

APPLICATION OF THE KRONEKE-KAPELLI THEOREM IN CHECKING THE SYSTEM  
OF LINEAR EQUATIONS

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**Abstract:** *The article presents two elements of the concept of a linear algebra lecture for economics studies. It attempts to demonstrate the significant role of ordering of the lectured content – with a focus on starting the lecture with systems of linear equations, and shows the considerable benefits of introducing the concept of the reduced row echelon form of matrix as one of the most useful concepts of linear algebra.*

**Keywords:** *linear algebra, reduced matrix, linear equations, economic studies, standards of teachings.*

### **1. Introduction**

The last few years have been a period of significant change in the teaching of mathematics and quantitative research methods at universities of economics (and all the others) in Poland. The reason lies in two main, mutually reinforcing factors – a significant reduction in the level of mathematics matriculation requirements and, simultaneously, decreasing the number of hours devoted to lectures on quantitative methods during studies. Attempts to clear up the confusion caused by these changes by adopting in 2005, the new “Law on Higher Education” 1 (standardization of names and curriculum content while reducing – by roughly one-third – the total number of hours devoted to lectures), did not produce the expected positive effects. As a result, from 1 October 2011, an amendment to the said Act entered into Piotr Dniestrzański 44 force 2 , which will also not cause a radical improvement of the situation. The most important change in the

amendment relates to more freedom to shape the curriculum for some (by definition more prestigious) universities. The

effects of the current six years of educational standards on the level of mathematics teaching in universities of economics are treated in the paper of Maciuk (2011). An analysis of teaching standards introduced in 2005, can be found in an article by Łyko (2007). In his article Dniestrzański (2011), conducted a discussion on the impact of changes in the education market in recent years, and the problems with the choice of field of study for high school graduates who are going to be educated in mathematical economics in a broad sense.

Systems of linear equations play a very important role in mathematics courses of all economics majors. The method of lecturing on this batch of material and its position in the course is often, in terms of the expected educational effects, not suitable. This is probably the result of underestimating the importance of this part of linear algebra in the understanding by the students of the next parts of the educational material. Problems with grasping this part of material have a definitely negative impact on the understanding of other subjects of linear algebra, which is a part of the mathematics course for all economics majors. This in turn causes problems in the reception of classes of some quantitative subjects such as econometrics, statistics

and forecasting and simulations.

*Reduced matrix.* The concept of the reduced matrix is very useful and relatively easily absorbed by the students. Mastering the matrix reduction techniques allows

it to be used in a number of algebraic problems. Below is the definition of an example of the reduced matrix. 3

**Definition.** Row Reduced matrix (or briefly, reduced matrix) is a matrix in which for each non-zero row there is at least one element, which is the only non-zero element in its column.

Example 1. Let

$$A = \begin{pmatrix} 0 & 0 & 3 & 0 \\ 0 & 8 & 0 & 4 \\ 7 & 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 2 & 4 & 0 & 0 \\ 0 & 8 & 0 & 4 \\ 0 & 0 & -5 & 0 \end{pmatrix}, C = \begin{pmatrix} -5 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 7 & 3 & 1 & 2 & 0 \end{pmatrix}$$

Matrices B and C are reduced, and A is not a reduced matrix. Elements referred to in definition 1 (which are the only non-zero elements in

their columns) have been marked. Note that in matrix  $A$  in the second row item 4 could also be marked – it is also the only non-zero element in its column.

*Systems of linear equations.* Solving systems of linear equations by reducing the augmented matrix is a natural and easily understood technique. It can be used even without a precise definition of the concept of the reduced matrix.

Example 2. Let us consider the system of equations

$$\begin{cases} 3x + 2y - 6z = 3 \\ x - 3y - 2z = -10. \\ -x + y + 2z = 4 \end{cases}$$

*Determinants.* The

components of matrix reduction may be used in the practice of calculating square matrices determinants, especially if the students have already mastered the technique of reduction. Take the following example.

Example 3. (The arrow indicates the column for which the Laplace

$$\begin{aligned} & \begin{vmatrix} 3 & 5 & \boxed{1} & 0 \\ 7 & 0 & 2 & -3 \\ 2 & 2 & 4 & 7 \\ -2 & 1 & 0 & 7 \end{vmatrix} \begin{matrix} w_2 - 2w_1 \\ = \\ w_3 - 4w_1 \end{matrix} \begin{vmatrix} 3 & 5 & \downarrow 1 & 0 \\ 1 & -10 & 0 & -3 \\ -10 & -18 & 0 & 7 \\ -2 & 1 & 0 & 7 \end{vmatrix} = \\ & = 1 \cdot (-1)^{1+3} \begin{vmatrix} 1 & -10 & -3 \\ -10 & -18 & 7 \\ -2 & \boxed{1} & 7 \end{vmatrix} \begin{matrix} w_1 + 10w_3 \\ = \\ w_2 + 18w_3 \end{matrix} = \\ & = 1 \cdot \begin{vmatrix} -19 & \downarrow 0 & 67 \\ -46 & 0 & 133 \\ -2 & 1 & 7 \end{vmatrix} = 1 \cdot (-1)^{3+2} \begin{vmatrix} -19 & 67 \\ -46 & 133 \end{vmatrix} = \\ & = (-1)(-19 \cdot 133 - 67 \cdot (-46)) = -555. \end{aligned}$$

transform is used)

The possibility of using the elements of matrix reduction for the calculation of the determinants is clear and is to be found in practically every linear algebra course. However, the use of the matrix reduction by students prior to solving systems of linear equations and testing linear independence of vectors, causes that, when reaching the notion of determinants (or successive elements of linear algebra where you can also use this technique), the ability is already known, mature and does not cause any difficulties.

**Conclusions**

Limiting the number of hours of classes for quantitative subjects in economics majors and limiting mathematics education in high school

enforces the concept of fundamental change in providing lectures and exercises in mathematics. The elements of linear algebra offered to students (more or less boiled down to those listed in Table 1) can be greatly enhanced using the concept of reduced matrix and the matrix reduction techniques. The presented examples of its applications have been used and successfully tested in practice in mathematics courses in various economics related studies at the Wrocław University of Economics. The degree

of mastery achieved by the students, that has been proved with regular tests and revised during final exams, shows that the concept is at least worthy of consideration. The specificity of economics studies in which mathematics is an auxiliary subject, puts great pressure on students to master the skill in the use of mathematical tools. The elements of the lecture concept of the algebraic part of the mathematics course can help to achieve this effect.

#### **REFERENCES:**

1. Antoniewicz R., Misztal A. (2007). *Matematyka dla studentów ekonomii. Wykłady z ćwiczeniami*. Wydawnictwo Naukowe PWN. Warszawa.
2. Dniestrzański P. (2011). *Studia ekonomiczno-matematyczne – analiza wybranych aspektów oferty edukacyjnej*. *Didactics of Mathematics*. No. 8 (12). Publishing House of the Wrocław University of Economics. Pp. 5-16.
3. Banaś J. (2005). *Podstawy matematyki dla ekonomistów*. Wydawnictwa Naukowo-Techniczne. Warszawa.
4. Bażańska T., Nykowska M. (2004). *Matematyka w zadaniach dla wyższych zawodowych uczelni ekonomicznych*. Oficyna Wydawnicza Branta. Bydgoszcz–Warszawa.
5. Bednarski T. (2004). *Elementy matematyki w naukach ekonomicznych*. Oficyna Ekonomiczna. Kraków. Piotr Dniestrzański 54
6. Łyko J. (2007). *O standardach kształcenia*. *Didactics of Mathematics*. No. 4 (8). Publishing House of the Wrocław University of Economics. Pp. 5-12.
7. Maciuk A. (2011). *Wpływ standardów kształcenia na poziom nauczania matematyki w wyższych szkołach ekonomicznych*. *Didactics of Mathematics*. No. 8 (12).
8. Publishing House of the Wrocław University of Economics. Pp. 81-90.
9. Piwecka-Staryszak A. (Ed.) (2004). *Wykłady z matematyki dla studentów uczelni*

10. ekonomicznych. Publishing House of the University of Wrocław.  
Wrocław.