FREDHOLMNING UMUMLASHGAN FUNDAMENTAL MUNOSOBATLARI

Tangirqulova Zebiniso Jasurovna
JDPU matematika o'qitish metodikasi yo'nalishi 2-kurs magistranti

Annotatsiya: *Kvant mexanikasi, qattiq jismlar nazariyasi va statistik fizika masalalarini yechish ko`p hollarda differensial va integral tenglamalar yechimlari xossalarini tadqiq qilish masalasiga keltiriladi. Differensial tenglamani yechish esa integral tenglamani yechish masalasiga keladi.*

Kalit so`zlar: *Fredholm umumlashgan fundamental munosobati, sonning kattaligi, simmetrik yadro, yechimlar sistemasi.*

Fredholmning umumlashgan fundamental munosabatlari quyidagilar:

$$D\begin{pmatrix} x_1, x_2, x_3, \dots & x_p \lambda \\ y_1, y_2, y_3, \dots & y_p \lambda \end{pmatrix} = \sum_{\alpha=1}^{p} (-1)^{\alpha+\beta} D\begin{pmatrix} x_1, \dots, x_{\alpha-1}, x_{\alpha+1}, & \dots, x_p, \lambda \\ y_1, \dots, y_{\beta-1}, y_{\beta+1} & \dots, y_p, \lambda \end{pmatrix} +$$

$$+\lambda \int_a^b K(t, y_\beta) D\begin{pmatrix} x_1, \dots, x_{\alpha-1}, x_{\alpha+1}, & \dots, x_p, \lambda \\ y_1, \dots, y_{\beta-1}, y_{\beta+1} & \dots, y_p, \lambda \end{pmatrix} dt \qquad (9)$$

$$D\begin{pmatrix} x_1, x_2, x_3, \dots & x_p \lambda \\ y_1, y_2, y_3, \dots & y_p \lambda \end{pmatrix} = \sum_{\alpha=1}^{p} (-1)^{\alpha+\beta} \lambda K(x_\alpha, y_\beta) D\begin{pmatrix} x_1, \dots, x_{\alpha-1}, x_{\alpha+1}, & \dots, x_p, \lambda \\ y_1, \dots, y_{\beta-1}, y_{\beta+1} & \dots, y_p, \lambda \end{pmatrix} +$$

$$+\lambda \int_a^b K(x_a, t) D\begin{pmatrix} x_1, \dots, x_{\alpha-1}, x_{\alpha+1}, & \dots, x_p, \lambda \\ y_1, \dots, y_{\beta-1}, y_{\beta+1} & \dots, y_p, \lambda \end{pmatrix} dt \qquad (10)$$

Yuqorida keltirilgan (7) munosabat quyidagi umumiy munosabatning xususiy holidir

$$\int_a^b \dots \int_a^b D\begin{pmatrix} x_1, x_2, x_3, \dots & x_p \lambda \\ y_1, y_2, y_3, \dots & y_p \lambda \end{pmatrix} dx_1, \dots dx_p = (-1)^p \lambda^p \Delta^{(p)}(\lambda) \qquad (11)$$

Faraz qilaylik, λ_0 soni $\Delta(\lambda) = 0$ tenglamaning ildizi bo`lsin. Ma'lumki,

$\Delta(0) = 1$ shuning uchun $\lambda_0 \neq 0$ $\Delta(\lambda)$ analitik funksiya bo`lganligi uchun $\lambda_0$ uning chekli r karrali noli bo`ladi, ya'ni

$$\Delta(\lambda_0) = 0, \quad \Delta'(\lambda_0) = 0, \ \dots, \ \Delta^{(r-1)}(\lambda_0) = 0, \ \Delta^{(r)}(\lambda_0) = 0$$

Agar biz (11) formulada $\lambda = \lambda_0$ va $p = r$ desak, u holda (11) ning o'ng tomoni nolmas bo`ladi. Demak, uning chap tomoni ham nolmas, bu esa o`z navbatida

p – tartibli $D_p(x,t;\lambda_0)$ minorning aynan nolmas ekanligini keltirib chiqaradi. Bu yerdan $D_p(x,t;\lambda_0)$ ning aynan nol funksiya emasligi kelib chiqadi. Agar λ_0 soni $\Delta(\lambda)$ funksiyaning r karrali noli bo`lsa, u holda shunday $q \leq r$ natural son mavjudki, quyidagilar bajariladi:

$$\Delta(\lambda_0) = 0, \ D(x,t;\lambda_0) \equiv 0 \ , \dots, \ D_{q-1}(x,t;\lambda_0) \equiv 0$$

bo`lib, $D_p(x,t;\lambda_0)$ aynan nolmas bo`ladi.

2-ta'rif. Yuqorida aniqlangan q soniga λ_0 xarakteristik sonning karraligi deyiladi.

Shuni ta'kidlaymizki, simmetrik yadrolar uchun $q = r$ tenglik o`rinli. Xususan bizning holimizda ham $q = r$ bo`ladi.

$D_p(x,t;\lambda_0)$ aynan nolmas funksiya bo`lganligi uchun shunday $x_1 = x'_1$, $x_2 = x'_2$, .... , $x_q = x'_q$, $y_1 = y'_1$, $y_2 = y'_2$, ....., $y_q = y'_q$ nuqtalar mavjud bo`lib,

$$D\begin{pmatrix} x'_1, x'_2, x'_3, \dots & x'_p\ \lambda_0 \\ y'_1, y'_2, y'_3, \dots & y'_p\ \lambda_0 \end{pmatrix} \neq 0$$

bo`ladi. Endi Fredholmning (10) umumlashgan fundamental munosabatida $\lambda = \lambda_0$ va $p = q$ va

$$x_1 = x'_1,\ x_2 = x'_2, \dots, x_{\alpha-1} = x'_{\alpha-1}, x_{\alpha+1} = x'_{\alpha+1}, \dots, x_q = x'_q,$$

$$y_1 = y'_1,\ y_2 = y'_2, \dots, y_{\alpha-1} = y'_{\alpha-1}, y_{\alpha+1} = y'_{\alpha+1}, \dots, y_q = y'_q$$

desak, quyidagi tenglikka ega bo`lamiz

$$D\begin{pmatrix} x'_1, x'_2, x'_3, \dots & x'_q\ \lambda_0 \\ y'_1, y'_2, y'_3, \dots & y'_q\ \lambda_0 \end{pmatrix} = \lambda_0 \int_a^b K(x,t) D\begin{pmatrix} x'_1, x'_2, x'_3, \dots & x'_q\ \lambda_0 \\ y'_1, y'_2, y'_3, \dots & y'_q\ \lambda_0 \end{pmatrix} dt \quad (12)$$

(12) tenglikning ikkala qismini noldan farqli bo`lgan

$$D\begin{pmatrix} x'_1, x'_2, x'_3, \dots & x'_q\ \lambda_0 \\ y'_1, y'_2, y'_3, \dots & y'_q\ \lambda_0 \end{pmatrix} = D_q(x', y'; \lambda_0)$$

bo`lamiz va

$$\varphi_0(x,\lambda_0) = \frac{D\begin{pmatrix} x'_1, x'_2, x'_3, \dots & x'_q\ \lambda_0 \\ y'_1, y'_2, y'_3, \dots & y'_q\ \lambda_0 \end{pmatrix}}{D_q(x', y'; \lambda_0)} \quad (13)$$

belgilash kiritib, barcha $\alpha = 1, 2, \dots, q$ larda quyidagiga ega bo`lamiz

$$\varphi_0(x,\lambda_0) = \lambda_0 \int_a^b K(x,t)\varphi_a(t,\lambda_0)dt \quad (14)$$

(14) tenglik $\varphi_1(x,\lambda_0)$, $\varphi_2(x,\lambda_0)$, , $\varphi_q(x,\lambda_0)$ lar bir jinsli $u(x) = \lambda_0 \int_a^b K(x,t)u(t)dt$ tenglamaning yechimlari ekanligini bildiradi. Bu yechimlar uzluksiz va (13) ga ko`ra

$$\varphi_\alpha(x'_\beta, \lambda_0) = \begin{cases} 1, agar \alpha = \beta \\ 0, agar \alpha \neq \beta \end{cases} \quad (15)$$

1-lemma. Bir jinsli $u(x) = \lambda_0 \int_a^b K(x,t)u(t)dt$ tenglamaning yechimlari sistemasi $\varphi_1(x,\lambda_0)$, $\varphi_2(x,\lambda_0)$, , $\varphi_q(x,\lambda_0)$ hiziqli erklidir.

Isbot. Faraz qilaylik,

$$C_1\varphi_1(x,\lambda_0)+C_2\varphi_2(x,\lambda_0)+\ldots+C_q\varphi_q(x,\lambda_0)=0$$

tenglik biror $C_1, C_2, C_3,\ldots,C_q$ sonlar uchun o`rinli bo`lsin. So`nggi tenglikda $x=x'_\alpha$ desak, (15) ga ko`ra $C_\alpha=0$, $\alpha=1,2,3,\ldots.q$ ga ega bo`lamiz.

Ma'lumki bir jinsli tenglama yechimlari yig`indisi va songa ko`paytmasi yana yechim bo`ladi. Shuning uchun

$$u(x)=C_1\varphi_1(x,\lambda_0)+C_2\varphi_2(x,\lambda_0)+\ldots+C_q\varphi_q(x,\lambda_0)=0 \qquad (16)$$

funksiya ixtiyoriy $C_1, C_2, C_3,\ldots,C_q$ sonlar uchun $u(x)=\lambda_0\int_a^b K(x,t)u(t)dt$ bir jinsli tenglamaning yechimi bo`ladi. Endi $u(x)=\lambda_0\int_a^b K(x,t)u(t)dt$ bir jinsli tenglamaning ixtiyoriy yechimi (16) ko`rinishga ega ekanligini ko`rsatamiz. Faraz qilaylik, $u(x)$ bir jinsli $u(x)=\lambda_0\int_a^b K(x,t)u(t)dt$ tenglamaning biror yechimi bo`lsin, ya'ni

$$u(t)-\lambda_0\int_a^b K(t,s)u(s)ds\equiv 0 \qquad (17)$$

bo`lsin. U holda ixtiyoriy $H(x\,t)$ uzluksiz funksiya uchun quyidagi ayniyat o`rinli

$$\int_a^b\left\{u(t)H(x,t)-\lambda_0\int_a^b K(t,s)u(s)H(x,t)ds\right\}dt\equiv 0 \qquad (18)$$

(17) dan (18) ni ayirib, quyidagiga ega bo`lamiz

$$u(x)=\lambda_0\int_a^b N(x,t)u(t)dt \qquad (19)$$

bu yerda

$$\lambda_0 N(x,t)=\lambda_0 K(x,t)-H(x,t)+\lambda_0\int_a^b K(s,t)H(x,s)ds$$

Endi Fredholmning (9) umumlashgan fundamental munosabatida $\lambda=\lambda_0$ va

$p = q+1$ va $x_{q+1}=x$, $y_{q+1}=y$ desak va x_i bilan x_j ning o'rni almashganda $D_p(x,y;\lambda_0)$ ning ishorasi almashinishini hisobga olsak, quyidagiga ega bo`lamiz

$$D\begin{pmatrix}x,x_{1,},x_2,x_3,\ldots & x_p\lambda_0\\ y,y_1,y_2,y_3,\ldots & y_p\lambda_0\end{pmatrix}=\lambda_0 K(x,y)D\begin{pmatrix}x_{1,},x_2,x_3,\ldots & x_p\lambda_0\\ y_1,y_2,y_3,\ldots & y_p\lambda_0\end{pmatrix}-$$

$$-\sum_{\alpha=1}^{q}\lambda_0 K(x_\alpha,y)D\begin{pmatrix}x_{1,},x_2,x_3,\ldots & x_p\lambda_0\\ y_1,y_2,y_3,\ldots & y_p\lambda_0\end{pmatrix}+ \qquad (20)$$

$$+\lambda_0\int_a^b K(s,y)D\begin{pmatrix}x_{1,},x_2,x_3,\ldots & x_p\lambda_0\\ y_1,y_2,y_3,\ldots & y_p\lambda_0\end{pmatrix}ds$$

(20) tenglikda

$$x_1 = x'_1\ ,\ x_2 = x'_2, ..., x_{\alpha-1} = x'_{\alpha-1}, x_{\alpha+1} = x'_{\alpha+1},....\ ,\ x_q = x'_q\ ,$$

$$y_1 = y'_1,\ y_2 = y'_2, ..., y_{\alpha-1} = y'_{\alpha-1}, y_{\alpha+1} = y'_{\alpha+1}\ ,......, y_q = y'_q$$

almashtirish qilamiz, hamda (20) tenglikning ikkala qismini noldan farqli bo`lgan

$$D\begin{pmatrix} x'_{1,}, x'_2, x'_3,... & x'_q\ \lambda_0 \\ y'_1, y'_2, y'_3,... & y'_q\ \lambda_0 \end{pmatrix} = D_q(x', y'; \lambda_0)$$

ga bo`lamiz va

$$H(x,y) = \frac{D\begin{pmatrix} x'_{1,}, x'_2, x'_3,... & x'_q\ \lambda_0 \\ y'_1, y'_2, y'_3,... & y'_q\ \lambda_0 \end{pmatrix}}{D_q(x', y'; \lambda_0)} \qquad (21)$$

belgilash kiritib quyidagiga ega bo`lamiz:

$$\sum_{\alpha=1}^{q} \lambda_0 K(x'_\alpha, t)\varphi_\alpha(x, \lambda_0) = \lambda_0 K(x,t) - H(x,t) + \lambda_0 \int_a^b K(s,t)H(x,s)ds \qquad (22)$$

(22) tenglikning o'ng tomoni $\lambda_0 N(x,t)$ ga teng. (18) aytiyat ixtiyoriy *H(x,t)* uzluksiz funksiya uchun o`rinli edi. Shuning uchun biz uni (21) tenglik bilan aniqlangan *H(x,t)* bilan almashtiramiz. Natijada

$$\lambda_0 N(x,t) = \sum_{\alpha=1}^{q} \lambda_0 K(x'_\alpha, t)\varphi_\alpha(x, \lambda_0)$$

tenglikni olamiz. $\lambda_0 N(x,t)$ ning bu ifodasini (19) tenglikning o`ng tomoniga qo`yib,

$$u(x) = \lambda_0 \sum_{\alpha=1}^{q} \varphi_\alpha(x, \lambda_0) \int_a^b K(x'_\alpha, t)\varphi_\alpha(x, \lambda_0)$$

tenglikka ega bo`lamiz. Bundan *u(x)* ning (16) ko`rinishda tasvirlanishi kelib chiqadi. Shunday qilib, biz Fredholmning ikkinchi fundamental teoremasini isbotladik.

4-teorema. Agar $\lambda = \lambda_0$ soni *K (x, t)* yadroning q karrali xarakteristik soni bo`lsa, u holda $u(x) = \lambda_0 \int_a^b K(x,t)u(t)dt$ bir jinsli tenglama q ta chiziqli bog'lanmagan

$$\varphi_\alpha(x, \lambda_0),\ \ \alpha = 1,2,3,....,q$$

yechimlarga ega bo`ladi va ixriyoriy u(x) yechim ularning chiziqli kombinatsiyasi ko`rinishida tasvirlanadi, ya'ni u(x) yechim uchun (16) tenglik o'rinli.

Bu chiziqli bog`lanmagan $\varphi_\alpha(x, \lambda_0),\ \ \alpha = 1,2,3,....,q$ yechimlar sistemasi (13) tenglik bilan aniqlanadi.

FOYDALANILGAN ADABIYOTLAR RO'YXATI:

1. J.I. Abdullayev, R.N. G`anixo`jayev, M.H. Shermatov, O.I.Egamberdiyev. Funksional analiz. O`quv qo`llanma. Toshkent-Samarqand. 2015.
2. J.I. Abdullayev, K.Kuliyev, I.Xayrullayev. Integral tenglamalar. Uslubiy qo'llanma. Samarqant. 2020.
3. Sarimsoqov T.A. Haqiqiy o`zgaruvchining funksiyalari nazariyasi. Toshkent: Fan. 1994.
4. Бахвалов Н.С., Жидков Н.П., Кобельков Г.Н. Численные методы. М: Наука, 1987. (571-581 betlar)
5. Воробьева Г.К., Данилова А.Н. Практикум по вычислительной математике. М: Высшая школа, 1990. (292-300 betlar)
6. Sarimsoqov T.A. Funksional analiz kursi. Toshkent: O`qituvchi. 1986.
7. Sh.O. Alimov, R.R.Ashurov. Matematik tahlil. 1-qism. Toshkent. Kamalak.2012.
8. Sh.A.Ayupov, M.A.Berdiqulov, R.M.Turg`unboyev. Funksiyalar nazariyasi. Toshkent.2004.
9. Haim Brezis. Functional Analysis, Sobolev Spaces and Partial Differenrial Equations. Springer, 2011
10. Sh.A.Ayupov, M.A.Berdiqulov, R.M.Turg`unboyev. Funksional analiz. Toshkent. 2008.