SUPER MATEMATIKADA GOLOMORF FUNKSIYALAR

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Annotatsiya: *Ushbu maqolada Matematika fanining asosiy yordamchisi hisoblanadi. Bu maqolani avzali tomonlari shundaki koshi teoremasi, koshining integral formulasi va funksiyani Teylor qatoriga yoyish keltirib o`tilgan. Golomorf funksiyaning istalgan tartibli hosilaga ega bo'lishi haqida yangi elementlarga ega bo`lishga ushbu maqola yordam beradi.*

Kalit so'zlar: *Koshi teoremasi, integral, Golomorf funksiya, orttirma, Teylor qator,*

Annotation: *This article is the main assistant in Mathematics. The advantages of this article are that Cauchy's theorem, Cauchy's integral formula, and the extension of the function to Taylor's series have been cited. This article will help to have new elements about whether a holomorphic function has any ordered derivative.*

Key words: *Koshi's theorem, integral, holomorphic function, gain, Taylor series,*

Аннотация: *В этой статье основное внимание уделяется математике. Изюминкой этой статьи является то, что теорема Коши, интегральная формула Коши и разложение функции на ряд Тейлора цитируется. Эта статья поможет вам получить новые элементы о том, как голоморфная функция может иметь производную желаемого порядка.*

Ключевые слова: *Теорема Коши, Интеграл, Голоморфная функция, вычитание, ряд Тейлора.*

Koshi teoremasi. Agar $f(z)$ funksiya bir bog'lamli D sohada $(D \subset C_z)$ golomorf bo'lsa, u holda $f(z)$ funksiyaning D sohada yqtuvchi har Qanday silliq (bo'lakli silliq) γ yopiq chiziq (yopiq kontur) bo'yicha integral nolga teng bo'ladi:

$$\int_{\gamma} f(z)dz = 0$$

Koshining integral formulasi.

Agar $f(z) \in \vartheta(D)$ $(D \subset C_z)$ va $\overline{D}$ da uzluksiz bo'lsa, u holda $\forall z \in D$ uchun

$$f(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{f(\xi)}{\xi - z} d\xi$$

tenglik o'rinli bo'ladi.

Golomorf funksiyaning istalgan tartibli hosilaga ega bo'lishi.

Agar $f(z)\in\vartheta(D)$ $\left(D\subset C_z\right)$ bo'lsa, u holda $f(z)$ D sohada istalgan tartibdagi hosilaga ega bo'lib ,

$$f^{n}(z)=\frac{n!}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{(\xi-z)^{n+1}}d\xi \quad \left(n=1,2,3...\right) \quad (1)$$

bo'ladi.

Bu yerda $\gamma-D$ sohada yotuvchi (bo'lakli silliq) yopiq chiziq bo'lib, z esa γ chiziq bilan chegaralangan sohaga tegishli nuqta .

Isbot. Koshining integral formulasiga ko'ra

$$f(z)=\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{\xi-z}d\xi$$

bo'ladi.

z nuqtaga Δz orttirma berib, $f(z)$ funksiya orttirmasini topamiz:

$$f(z+\Delta z)\text{ - }f(z)=\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)d\xi}{\xi-z-\Delta z}-\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{\xi-z}d\xi=$$

$$\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}f(\xi)\left(\frac{1}{\xi-z-\Delta z}-\frac{1}{\xi-z}\right)d\xi=\frac{\Delta z}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)d\xi}{(\xi-z-\Delta z)(\xi-z)} .$$

Unda

$$\frac{f(z+\Delta z)-f(z)}{\Delta z}=\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{(\xi-z-\Delta z)(\xi-z)}d\xi$$

bo'ladi. Keyingi tenglikni quyidagicha yozib olamiz:

$$\frac{f(z+\Delta z)-f(z)}{\Delta z}=\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{(\xi-z)^{2}}d\xi+\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{(\xi-z-\Delta z)(\xi-z)}d\xi-$$

$$-\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{(\xi-z)^{2}}d\xi=\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{f(\xi)}{(\xi-z)^{2}}d\xi+\frac{1}{2\pi\,\mathrm{i}}\int_{\gamma}\frac{\Delta z f(\xi)d\xi}{(\xi-z-\Delta)(\xi-z)^{2}}. \qquad (2)$$

Endi

$$\frac{1}{2\pi i}\int_{\gamma}\frac{\Delta z f(\xi)}{(\xi-z-\Delta z)(\xi-z)^{2}}d\xi$$

integralni baholaymiz. Ravshanki,

$$\left|\frac{1}{2\pi i}\int_{\gamma}\frac{\Delta z f(\xi)}{(\xi-z-\Delta z)(\xi-z)^{2}}d\xi\right|<\frac{|\Delta z|}{2\pi}M\int_{\gamma}\frac{|d\xi|}{|\xi-z-\Delta z||\xi-z|^{2}}$$

bunda

$$M=\max_{\gamma}\left|f(t)\right|.$$

Agar z nuqtadan γ chiziqgacha bo'lgan masofani $2d$ $(d>0)$ desak, unda

$$|\xi - z| > d,\ |\xi - z - \Delta z| > d$$

bo'lib, (agarda $|\Delta z|$ etarlicha kichiq bo'lsa)

$$\left|\frac{1}{2\pi i}\int_{\gamma}\frac{\Delta z f(\xi)}{(\xi - z - \Delta z)(\xi - z)^2}d\xi\right| < \frac{|\Delta z| Ml}{2\pi d^2} \qquad (3)$$

bo'ladi. Bu erda $l-\gamma$ chiziq uzunligi.

(1) ni e'tiborga olib, $\Delta z \to 0$ da (2) da limitga o'tib

$$f'(z) = \frac{1}{2\pi i}\int_{\gamma}\frac{f(\xi)}{(\xi - z)^2}d\xi$$

bo'lishini topamiz.

Endi $f'(z)$ funksiyani olib uning uchun yuqoridagi mulohazalarni takrorlasak

$$f''(z) = \frac{2!}{2\pi i}\int_{\gamma}\frac{f(\xi)}{(\xi - z)^3}d\xi \qquad (4)$$

tenglik hosil bo'ladi.

Xuddi shu yo'l bilan uchinchi, to'rtinchi va hakozo tartibdagi hosilalarni mavjudligi ko'rsatiladi. $f(z)$ funksiyaning n–tartibli $(n=3,4,5,\ldots)$ hosilasi uchun (1) ni o'rinli bo'lishi matematik induksiya usuli yordamida isbotlanadi.

Natija 1. Agar $f(z) \in \vartheta(D)$ $(D \subset \mathbf{C}_z)$ bo'lsa, $f'(z) \in \vartheta(D)$ bo'ladi.

Natija 2. Agar $f(z)$ funksiya D sohada $(D \subset \mathrm{C}_z)$ boshlang'ich funksiyaga ega bo'lsa, u holda $f(z)$ D sohada golomorf bo'ladi.

Funksiyani Teylor qatoriga yoyish.

Agar $f(z) \in \vartheta(D)$ $(D \subset \mathbf{C}_z)$ bo'lsa, u holda $a \in D$ nuqtada (a nuqtaning

$$\bigcup_{\rho}(a) = \{z \in \mathbf{C}_z : |z-a| < \rho, \rho > 0\} \subset D$$

atrofida) Teylor qatoriga yoyiladi:

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n = \sum_{n=0}^{\infty}\frac{f^{n}(a)}{n!}(z-a)^n$$

Isbot. $\bigcup_{\rho}(a)$ ning chegarasini γ deylik.

$$\gamma = \{z \in \mathbf{C}_z : |z-a| = \rho,\ \rho > 0\}$$

bo'ladi.

Avvalo $\dfrac{1}{\xi - z}$ funksiyani quyidagicha

$$\frac{1}{\xi - z} = \frac{1}{\xi - a - (z - a)} = \frac{1}{(\xi - z)\left(1 - \frac{z-a}{\xi - a}\right)}$$

yozib, so'ng

$$\frac{1}{1 - \frac{z-a}{\xi - a}} = \sum_{n=0}^{\infty} \left(\frac{z-a}{\xi - a}\right)^n$$

bo'lishini e'tiborga olib topamiz:

$$\frac{1}{\xi - z} = \sum_{n=0}^{\infty} \left(\frac{z-a}{\xi - a}\right)^n . \qquad (6)$$

Bu geometrik qator bo'lib, uning maxraji $\frac{z-a}{\xi - a}$ ga teng.

Ravshanki, $\xi \in \gamma$ uchun quyidagi tengsizlik

$$\left|\frac{z-a}{\xi - a}\right| = \frac{|z-a|}{\rho} = q < 1$$

o'rinli. Demak, (4) qator yaqinlashuvchi.

(6) tenglikning har ikki tomonini $\frac{1}{2\pi i} f(\xi)$ ga ko`paytirib, so'ng γ chiziq bo'yicha integrallab, ushbu

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f(\xi)}{\xi - z} d\xi = \frac{1}{2\pi i} \int_{\gamma} \sum_{n=0}^{\infty} \frac{f(\xi)}{(\xi - a)^{n+1}} (z-a)^n d\xi$$

tenglikka kelamiz.

(5) va (6) munosabatlardan

$$f(z) = \frac{1}{2\pi i} \int_{\gamma} \sum_{n=0}^{\infty} \frac{f(\xi)}{(\xi - a)^{n+1}} (z-a)^n d\xi$$

bo'lishi kelib chiqadi.

Integral ostidagi

$$\sum_{n=0}^{\infty} \frac{f(\xi)}{(\xi - a)^{n+1}} (z-a)^n$$

qatorning hadlari uchun

$$\left|\frac{f(\xi)}{(\xi - a)^{n+1}} (z-a)^n\right| < \frac{1}{\rho} M q^n \quad (n = 1,2,\ldots) \quad \left(M = \max_{\gamma} |f(\xi)|\right)$$

tengsizlik o'rinli bo'ladi.

Ravshanki,

$$\sum_{n=0}^{\infty} \frac{M}{\rho} q^n \qquad (q<1)$$

qator yaqinlashuvchi. Unda Veyershtrass alomatiga ko'ra

$$\sum_{n=0}^{\infty} \frac{f(\xi)}{(\xi-a)^{n+1}} (z-a)^n$$

funktsional qator γ da tekis yaqinlashuvchi bo'ladi. Binobarin, bu qatorni hadlab integrallash mumkin. Unda (7) tenglik ushbu

$$f(\xi)=\sum_{n=0}^{\infty}\left[\frac{1}{2\pi i}\int_{\gamma}\frac{f(\xi)}{(\xi-a)^{n+1}}d\xi\right](z-a)^n$$

ko'rinishga keladi. Yuqorida keltirilgan ma'lum teoremaga ko'ra

$$C_n=\frac{1}{2\pi i}\int_{\gamma}\frac{f(\xi)}{(\xi-a)^{n+1}}d\xi=\frac{f^{(n)}(a)}{n!}$$

bo'lishini topamiz. Natijada (8) va (9) tengliklardan

$$f(\xi)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(z-a)^n=\sum_{n=0}^{\infty}C_n(z-a)^n$$

bo'lishi kelib chiqadi. Bu esa $f(z)$ funksiyani Teylor qatoriga yoyilganini bildiradi.

Natija 3. Agar $f(z)$ funksiya yopiq doirada golomorf bo'lib, bu doiraning chegarasi $\gamma=\partial U_\rho(a)$ aylanada

$$|f(z)|\le M \quad (M-const)$$

bo'lsa, u holda $f(z)$ funksiya Teylor qatorining C_n koeffitsentlari uchun

$$|C_n|\le\frac{M}{\rho^n} \quad (n=0,1,2,\ldots)$$

tengsizlik o'rinli bo'ladi.

Haqiqatan ham, (9) formuladan

$$|C_n|\le\left|\frac{1}{2\pi i}\int_{\gamma}\frac{f(t)}{(t-a)^{n+1}}dt\right|\le\frac{1}{2\pi}\int_{\gamma}\frac{|f(t)|}{|t-a|^{n+1}}\cdot dt\le\frac{1}{2\pi}\cdot\frac{M}{\rho^{n+1}}\cdot 2\pi\rho=\frac{M}{\rho^n}$$

bo'lishi kelib chiqadi.

Odatda (10) tengsizllik Koshi tengsizligi deyiladi.

Liuvil teoremasi.

Agar $f(z)\in\vartheta(C)$ bo'lib, u chegaralangan bo'lsa, $f(z)$ funksiya C da o'zgarmas bo'ladi.

Isbot. Golomorf funksiyaning xossasiga ko'ra, $f(z)$ funksiya $|z-a|<\rho$ doirada $z-a$ ning darajalari bo'yicha Teylor qatoriga yoyiladi:

$$f(z)=\sum_{n=0}^{\infty}C_n(z-a)^n, \text{ bunda } C_n=\frac{1}{2\pi i}\int_{\gamma}\frac{f(z)}{(\xi-a)^{n+1}}d\xi.$$

Koshi tengsizligi (10) ga binoan $|f(z)|\le M$ bo'ladi. $f(z)\in\vartheta(C)$ bo'lgani uchun bu tengsizlikda ρ ni istalgancha katta qilib olish mumkin. Shuning uchun $n=1,2,3,\dots$ bo'lganda

$$\lim_{\rho\to\infty}\frac{M}{\rho^n}=0 \quad (n=1,2,3,..)$$

bo'ladi. Ayni paytda (10) tengsizlikning chap tomoni ρ ga boglik emas. Binobarin $n=1,2,3,\dots$ bo'lganda

$$C_n=0 \ (n=1,2,3,\dots)$$

bo'ladi. Demak, C da $f(z)=c_0 \qquad (c_0=const)$.

Morera teoremasi. Faraz qilaylik, $f(z)$ funksiya bir bog'lamli D sohada $D\subset C_z$ aniqlangan va uzluksiz bo'lib, γ esa shu D sohada yotuvchi ixtiyoriy silliq (bo'lakli silliq) yopiq chiziq bo'lsin. Agar $\int_{\gamma}f(z)dz=0$ bo'lsa, u holda $f(z)$ funksiya D sohada golomorf bo'ladi.

Isbot. Teoremada keltirilgan shart bajarilganda funksiya D sohada boshlang'ich $F(z)$ funksiyaga ega bo'lib, $F(z)$ funksiya D da C differensiallanuvchi, ya'ni golomorf bo'ladi.

3^0-xossaning 1-natijasiga ko'ra $F'(z)$ ham D sohada golomorf bo'ladi. Ayni paytda

$$F'(z)=f(z)$$

bo'lganligi sababli $f(z)\in\vartheta(D)$ bo'ladi.

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