KOMPLEKS SONLAR

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Annotatsiya: *Ushbu maqolada oliy matematikaning asosiy bo`limlaridan birir kampleks sonlar tahlili qilingan. Oliy matematikaga kerak boladigon tenglamalarining haqiqiy ildizlaridan tashqari ko`mpleks ildzlarini to`pishning oson usullari ko`rsatib otilgan. Kompleks sonlarning triganametrik shakli , geometrik tasviri, ildizdan chiqarish va kompleks sonning lagarifimi malumot berilgan va bu malimotlarga alohida misollar to`plami ham berilib otilgan.*

Kalit so'zlar: *Ta'rif , isbot , son, mavhum son, mavhum birlik,triganametrik shakl, kompleks tekislik, kompleks son, kvadrat ildiz, kompleks sonning logarifmi.*

Аннотация: *В этой статье представлен один из основных разделов высшей математикиразбор комплексных чисел. Высшая математика нуждается в Ребенок, показывая простые способы нахождения комплексных корней, отличных от действительных корней уравнений. Приведены тригонометрическая форма, геометрическое представление комплексных чисел, извлечение корня и лагарифм комплексного числа, а также приведен набор отдельных примеров этих ссылок.*

Ключевые слова*:* *Определение, доказательство, число,абстрактное число, абстрактная единица, триганаметрическая форма, комплексная плоскость, комплексное число, квадратный корень, логарифм комплексного числа.*

Annotation: *This article analyzes the birir kamplex numbers from the main sections of higher mathematics. Higher mathematics needs easy methods of collecting coomlex ildzs, in addition to the real roots of the child's equations, are shown. The triganametric form of complex numbers , the geometric representation, the derivation and the lagarithim of the complex number are reported, and these malimots are also given a separate set of examples.*

Key words: *Definition , proof, number, abstract number,abstract unit, triganametric form, complex plane, complex number, square root, logarithm of complex number.*

KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR

1-ta`rif. Kompleks son deb z=a+b$i$ ifodaga aytiladi. Bu yerda a, b$\in R$, $i=\sqrt{-1}$ bo`lib, mavhum birlik deb ataladi.

$i^2=-1, i^3=-i, i^4=1,\ldots, i^{4n}=1, i^{4n+1}=i, i^{4n+2}=-1, i^{4n+3}=-i, i=\sqrt{-1}$ soni biror real kattalikni ifodalamaydi. a – kompleks sonning haqiqiy qismi, bi – mav-hum qismi deyiladi. Kompleks sonning ma`nosi ham uning haqiqiy a va mavhum bi sonlar "kompleksidan" iborat ekanligidadir. z=a+bi kompleks sonning algebraik shakli deyiladi.

$a = \mathrm{Re}\, z$ va $b = \mathrm{Im}\, z$ deb belgilash qabul qilingan (Re fransuzcha reele – haqiqiy, Im – fransuzcha imaginaire – mavhum).

Agar (1) da b=0 bo`lsa $z = a$ haqiqiy son hosil bo`ladi, demak haqi-qiy sonlar to`plami $R$ kompleks sonlar to`plami C ning qism – to`p-lamidir. $R \subset C$

Agar a=0 bo`lsa, $z = bi$ sof mavhum son hosil bo`ladi, a=b=0 bo`l-ganda $z = 0$ kompleks son hosil bo`ladi.

2-ta`rif. Ikkita kompleks son z=a+bi va w=c+di teng deyiladi, agar a=c va b=d bo`lsa, ya`ni haqiqiy va mavhum qismlari mos ravishda teng bo`lsa, masalan: z=1,5+0,4i va $w = \frac{3}{2} + \frac{2}{5} i$ bo`lsa, z=w, chunki $1{,}5 = \frac{3}{2}$ va $0{,}4 = \frac{2}{5}$. Kompleks sonlar uchun katta yoki kichik munosabatlar aniqlanmaydi.

3-ta`rif. Bir-biridan faqat mavhum qismining ishorasi bilan farq qiluvchi ikkita kompleks son: $z = a + bi$ va $z$ =a-bi qo`shma kompleks sonlar deyiladi. z ga qo`shilgan sonni z bilan belgilash qabul qilingan: z =3+2i, $z = 3 - 2i$.

Haqiqiy son a ga qo`shmasi o`zi bo`ladi: $\overline{a} = \overline{a + 0 \times i} = a - 0 \times i = a$.

Kompleks sonlar ustida arifmetik amallar haqiqiy sonlar ustidagi amallarga o`xshaydi:

$$(a + bi) + (c + di) = (a + c) + (b + d)i \qquad (2)$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i \qquad (3)$$

$$(a + bi) \times (c + di) = (ac - bd) + (ad + bc)i \qquad (4)$$

$$\frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i \qquad (5)$$

Ko`rinadiki, kompleks sonlarning yig`indisi, ayirmasi, ko`paytmasi va bo`linmasi yana kompleks sondan iborat. (2) va (3) – amallarga bevo-sita ishonch hosil qilish mumkin. (4) va (5) ni keltirib chiqaramiz. $(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$, bu yerda i^2=-1 ekanligi hisobga olindi;

$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(a + di)(c - di)} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$ bo`lib, bundan (5) hosil bo`ladi.

(a+bi)+(a-bi)=2a;
(a+bi)×(a-bi)=a^2+b^2,

ya`ni qo`shma kompleks sonlarning yig`indisi va ko`paytmasi haqiqiy songa teng.

Misollar

$(3+4i)+(-2-5i)=1-i$

$(2+3i)-(-3+2i)=5+i$

$(2-5i)\cdot(4+i)=(8+5)+(2-20)i=13-18i$

$\frac{3-2i}{2+i}=\frac{(3-2i)(2-i)}{(2+i)(2-i)}=\frac{(6-2)+(-3-4)i}{4+1}=\frac{4}{5}-\frac{7}{5}i.$

Kompleks sonlar ustida arifmetik amallarning quyidagi xossalarini o`zingiz tekshirib ko`ring:

(Z, W va U – kompleks sonlar)

1) $Z+W=W+Z,$

2) $Z\times W=W\times Z,$

3) $(Z+W)+U=Z+(W+U),$

4) $(ZW)\times U=Z\times(WU),$

5) $Z\pm 0=Z,$

6) $Z\times 1=Z,$

7) $Z\times(W\pm U)=ZW\pm ZU;$

Agar Z va W kompleks sonlar Z +W=0 tenglikni qanoatlantirsa, va W o`zaro qarama qarshi kompleks sonlar deyiladi. $Z$ ga yagona qarama-qarshi son mavjud bo`lib, uni $-Z$ bilan belgilash qabul qilingan: $Z=2+5i$ ga qarama-qarshi son $-Z=-2-5i$ dir.

Agar Z va W kompleks sonlar $Z\times W=1$ tenglikni qanoatlantirsa, Z va W o`zaro teskari kompleks sonlar deyiladi. Har qanday $Z\neq 0$ kompleks songa yagona teskari son mavjud, bu son $\frac{1}{Z}$ bilan belgilanadi: $Z=2-3i$ ga teskari son: $\frac{1}{Z}=\frac{1}{2-3i}$ dan iborat. $Z=0$ songa teskari son mav-jud emas.

$Z=a+bi$ ga teskari sonni quyidagicha yozish maqsadga muvofiqdir:

$$\frac{1}{z}=\frac{1}{a+bi}=\frac{a-bi}{(a+bi)(a-bi)}=\frac{a-bi}{a^2+b^2}=\frac{a}{a^2+b^2}-\frac{b}{a^2+b^2}i$$

Kompleks songa teskari sonni topishda quyidagi teoremalardan foydalanish mumkin:

1-teorema: $\overline{Z+W}=\overline{Z}+\overline{W}$

2-teorema: $\overline{Z^n}=(\overline{Z})^n$

Kompleks sondan kvadrat ildiz chiqarish

Kompleks son z dan kvadrat ildiz deb, $w^2=z$ tenglikni qanoatlantiradigan har qanday kompleks songa aytiladi. Umuman z dan n-natural darajali ildiz deb, $w^n=z$ tenglamani qanoatlantiruvchi har qanday w kompleks songa aytiladi.

Agar z=0 bo`lsa, $w^n=0$ tenglik faqat w=0 uchun bajariladi. Agar z≠0 bo`lsa, $w^n=z$ tenglama n ta har xil qiymatlarda bajariladi:

Agar z=r(cosφ+isinφ) berilgan bo`lsa $w^n=z$ tenglamaning ildizlari quyidagi formuladan topiladi:

$$w_k=\sqrt[n]{r}\left(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n}\right),\qquad k=0,1,2,\dots,n-1.$$

Bu formulada k ga ketma-ket 0, 1, 2, ...n-1 qiymatlarni berib w_k uchun n ta har xil qiymat hosil qilamiz. k ning boshqa qiymatlarida w_k qiymati oldin hosil bo`lgan qiymatlaridan birortasiga teng bo`ladi.

Misol. $\sqrt{\sqrt{2}-i\sqrt{2}}$ ning barcha w_k (k=0, 1) qiymatlarini toping.

Yechish: $z=\sqrt{2}-i\sqrt{2}$ $\qquad r=|z|=\sqrt{2+2}=2$

$$\cos\varphi=\frac{\sqrt{2}}{2}; \qquad \sin\varphi=-\frac{\sqrt{2}}{2} \qquad demak \quad \varphi=\frac{7\pi}{4}$$

$$z=2(\cos(\frac{7\pi}{4})+i\sin(\frac{7\pi}{4})) \quad w_k=\sqrt{2}(\cos(\frac{\frac{7\pi}{4}+2\pi k}{2})+i\sin(\frac{\frac{7\pi}{4}+2\pi k}{2})) \qquad k=0 \ bo`lsa,$$

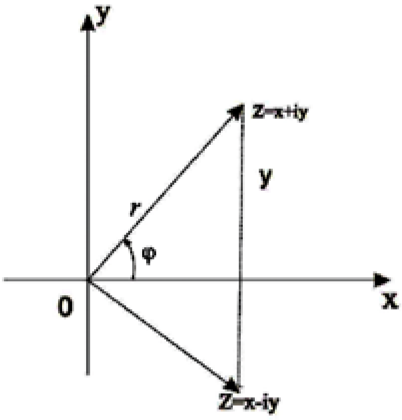
$$w_0=\sqrt{2}(\cos(\frac{7\pi}{8})+i\sin(\frac{7\pi}{8}))=\sqrt{2}(-\cos\frac{\pi}{8}+i\sin(-\frac{\pi}{8}))=\sqrt{2}\left(-\frac{\sqrt{2+\sqrt{2}}}{2}+i\frac{\sqrt{2-\sqrt{2}}}{2}\right)$$

$$k=1 \ bo`lsa \ w_1=\sqrt{2}(\cos(\frac{7\pi}{8}+\pi)+i\sin(\frac{7\pi}{8}+\pi))=\sqrt{2}\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)=\sqrt{2}\left(-\frac{\sqrt{2+\sqrt{2}}}{2}-i\frac{\sqrt{2-\sqrt{2}}}{2}\right)$$

Kompleks sonning geometrik tasviri va kompleks tekslik

To`g`ri burchakli Dekart koordinatalar sistemasi $x0y$ ni tanlab, uning abssissalar o`qiga $z=x+iy$ ning haqiqiy qismi x ni, ordinatalar o`qiga esa mavhum qismining koeffitsienti y ni joylashtirsak, tekislikda (x,y) nuqtaga ega bo`lamiz.

Ana shu nuqta $z=x+iy$ kompleks sonning geometrik tasviri deb qabul qilingan.



1-chizma

Shunday qilib, har bir kompleks songa tekislikda birgina nuqta va aksincha, tekislikdagi har bir nuqta uchun bitta kompleks son mos keladi.

$0x$ o`q – haqiqiy o`q, 0y – mavhum o`q, $x0y$ tekislik esa kompleks tekislik deyiladi.

Ko`pincha Z kompleks sonning geometrik tasviri sifatida koordinatalar boshini tekislikdagi Z nuqta bilan tutashtiruvchi vektor ham qabul qilinadi. Bu vektorning moduli yoki uzunligi: $|z|=\sqrt{x^2+y^2}$ (3.1)

Kompleks sonning trigonometrik va ko`rsatkichli shakli

1–chizmadan ko`rinadiki: $x = r\cos\varphi, y = r\sin\varphi$ (4.1). Bundagi r kompleks z sonni tasvirlagan vektorning uzunligini ifodalaydi, uni Z sonning moduli, φ burchakni esa Z ning argumenti deyiladi va u quyidagicha yoziladi:

$r = |z| = |a+ib| = \sqrt{a^2+b^2}$, $\arg Z = \varphi$ (4.2)

z kompleks songa mos bo`lgan vektorga birgina uzunlik va cheksiz ko`p burchaklar mos kelishi chizmadan ko`rinadi: $\varphi, \varphi+2\pi, \ldots$. Shu sababli odatda burchakning umumiy ko`rinishi $Argz = \arg z + 2\pi k$ (4.3) kabi belgilanib $(k = 0, \pm1, \pm2, \ldots)$, $\varphi = \arg Z$ ni argumentning bosh qiymati deyiladi.

Chizmadan: $tg\varphi = \frac{y}{x}$. Bunda

$$\varphi = \begin{cases} arctg\frac{y}{x}, agar..x \geq 0, y \geq 0 \\ \pi + arctg\frac{y}{x}, agar..x < 0, y \geq 0 \\ -\pi + arctg\frac{y}{x}, agar..x < 0, y < 0 \\ 2\pi + arctg\frac{y}{x}, agar..x \geq 0, y < 0 \\ \frac{\pi}{2}, agar..x = 0, y > 0 \\ -\frac{\pi}{2}, agar..x = 0, y < 0 \end{cases} \quad (4.4\ \ )$$

Endi (4.1) ga asosan $z = x + iy = r(\cos\varphi + i\sin\varphi)$ (4.5) bo`lib, o`ng tomon z kompleks sonning trigonometrik shakli (formasi) deyiladi. ($0 \leq r < \infty$ va $0 \leq \varphi < 2\pi$).

Matematik tahlildan Eylerning quyidagi mashhur formulasi ma`lum: $e^{i\varphi} = \cos\varphi + i\sin\varphi$ bunda $\varphi$-haqiqiy son. U holda (4.5) dan Z kompleks sonning ushbu ko`rsatkichli formasi

$z = re^{i\varphi}$ (4.6) kelib chiqadi, bunda $e = \lim_{x\to\infty}(1+\frac{1}{x})^x$, $2 < e < 3$, e=2.718281828459045...

2-misol. $z = i$ sonni trigonometrik va ko`rsatkichli shaklga keltiring.

Yechish. $x = 0, y = 1 > 0, r = |z| = \sqrt{0^2+1^2} = 1, \varphi = \frac{\pi}{2}$. (4.5)ga asosan $z = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

yoki (4.6) ga asosan: $z = i = e^{i\frac{\pi}{2}}$

Algebraik shaklda berilgan kompleks sonlarni darajaga ko`tarish va ildizdan chiqarish

Algebraik shaklda berilgan kompleks sonni n-darajaga ko`tarish uchun, uni avval trigonometrik shaklga keltirilib uning modulini shu darajaga ko`tarib, argumentini n ga ko`paytirish kerak:

$z^n = [r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi)$ (5.1) ga Muavr formulasi deyiladi.

3-misol $(1+i)^7$ ni hisoblang

Yechish. Dastlab qavslar ichidagi sonni trigonometrik shaklga keltirib olamiz: $1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$. Endi (5.1) formulaga asosan, buni darajada ko`tarib soddalashtiramiz:

$$(1+i)^7=\left(\sqrt{2}\right)^7\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)=8\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=8\sqrt{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=8(1-i)$$

$z=x+iy$ kompleks son berilgan bo`lsa, uning istalgan darajali ildizlarini topish bilan shug`ullanamiz.

Agar $z=\beta^n$ bo`lsa, $\beta$ soni z ning n-darajali ildizi deyilib $\beta=\sqrt[n]{z}$ (5.2) ko`rinishda yoziladi.

Biz mana shu β sonni topish uchun dastlab berilgan z sonni trigonometrik shaklga keltiramiz: $z=r(\cos\varphi+i\sin\varphi)$. Kompleks sonlar ustida to`rt amalni bajargan vaqtimizda yana kompleks sonlar hosil bo`lishini ko`rgan edik. Kompleks sonning ildizi ham kompleks son bo`ladi,ya`ni $\beta_k=\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right)$ (5.3), bunda k=0,1,2,3..., qiymatlarni qabul qilish mumkin.

Demak, algebraik formada berilgan kompleks sondan ildiz chiqarish uchun, avval uni trigonometrik shaklga keltirib, moduldan shu darajali ildiz chiqariladi, argumenti esa ildiz ko`rsatkichiga bo`linadi.

4. misol. $\sqrt[3]{1-i}$ ning qiymatlarini toping.

Yechish. Dastlab $1-i$ ni trigonometrik shaklga keltiramiz: $r=\sqrt{2}$ va $\varphi=-\frac{\pi}{4}$ bo`lgani uchun $1-i=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

$$\beta_k=\sqrt[3]{1-i}=\sqrt[3]{\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}=\sqrt[6]{2}\left(\cos\frac{\pi/4+2k\pi}{3}-i\sin\frac{\pi/4+2k\pi}{3}\right), k=\overline{0,2}.$$

k=0 da $\beta_0=\sqrt[6]{2}\left(\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}\right)$,

k=1 da $\beta_1=\sqrt[6]{2}\left(\cos\frac{3\pi}{4}-i\sin\frac{3\pi}{4}\right)$,

k=2 da $\beta_2=\sqrt[6]{2}\left(\cos\frac{17\pi}{12}-i\sin\frac{17\pi}{12}\right)$.

Trigonometrik va ko`rsatkichli shakldа berilgan kompleks sonlarni ko`paytirish, bo`lish, darajaga ko`tarish va ildiz chiqarish

Ushbu z_1 va z_2 kompleks sonlar berilgan bo`lsin.

1. Ko`paytirish.

$z_1=r_1(\cos\varphi_1+i\sin\varphi_1)=r_1e^{i\varphi_1}$, $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)=r_2e^{i\varphi_2}$, $z_1\cdot z_2=r_1e^{i\varphi_1}\cdot r_2e^{i\varphi_2}=$
$=r_1\cdot r_2e^{i(\varphi_1+\varphi_2)}$ (6.1)

Demak, kompleks sonlarni ko`paytirishda modullari ko`paytiriladi, argumentlari qo`shiladi.

2.Bo`lish.

$z_1 = r_1 e^{i\varphi_1}$ va $z_2 = r_2 e^{i\varphi_2}$ kompleks sonlar berilgan bo`lsin. $\frac{z_1}{z_2} = \frac{r_1 e^{i\varphi_1}}{r_2 e^{i\varphi_2}} = \frac{r_1}{r_2} e^{i(\varphi_1 - \varphi_2)} =$

$= \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)]$ (6.2)

Demak, trigonometrik formada berilgan kompleks sonlarni bo`lishda ularning argumentlari ayriladi, modullari bo`linadi.

3. Darajaga ko`tarish.

$z = re^{i\alpha}$ kompleks sonini n-darajaga ko`taraylik. $z^n = \left(re^{i\varphi}\right)^n = r^n e^{in\varphi}$ yoki $z^n = r^n(\cos n\varphi + i\sin n\varphi)$ (6.3).

Demak, trigonometrik formada berilgan kompleks sonni darajaga ko`tarishda modul va argument ham shu darajaga ko`tariladi.

Agar (6.3) da $r = 1$ bo`lsa, $[r(\cos\varphi + i\sin\varphi)]^n = \cos n\varphi + \sin n\varphi$ Muavr formulasi hosil bo`ladi.

4. Ildiz chiqarish.

$z = re^{i\varphi}$ kompleks sonning n-darajali ildizi $w = \rho(\cos\varphi + i\sin\varphi) = \rho e^{i\varphi}$ bo`lsa, ya`ni $\sqrt[n]{z} = w = \rho e^{i\varphi}, z = w^n = [\rho(\cos\varphi + i\sin\varphi)]^n = \rho^n(\cos n\varphi + \sin n\varphi)$,

$\cos\varphi = \cos n\varphi, \sin\varphi = \sin(\varphi + 2k\pi)$ uchun, $r = \rho^n, n\varphi = \varphi + 2k\pi, \rho = \sqrt[n]{r}, \varphi = \frac{\varphi + 2k\pi}{n}$,

$\sqrt{z} = \sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left[\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right]$ (6.4)

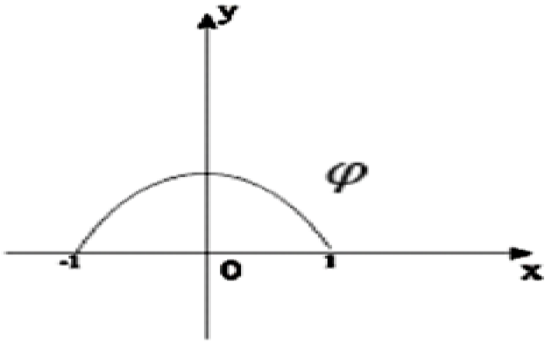
Demak, trigonometrik formada berilgan kompleks sondan ildiz chiqarish uchun, moduldan shu darajali ildiz chiqariladi, argumenti esa ildiz ko`rsatkichiga bo`linadi.

Kompleks sonning logarifmi

$z = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$ kompleks son berilgan bo`lsin. $z = re^{i\varphi}, z = re^{i(\varphi + 2k\pi)}$, $\ln z = \ln r + i\varphi \ln e = \ln r + i\varphi$ (7.1)

$\ln z = \ln r + i\varphi + 2k\pi$ (7.2)

5-misol.



2-chizma

$z = -1$ ning logarifmini toping.

Yechish. $z = -1 = \cos\pi + i\sin\pi$, $r = 1$, $\varphi = \pi$

$\ln z = \ln 1 + i\pi = i\pi$, $\ln z = i\pi + 2k\pi i = i\pi(1+2k)$, $k = 0, \pm1, \pm2.....$

Mashqlar

1. $\sqrt{z}$ ni hisoblang, agar:

1) $z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

2) $z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$,

3) $z = \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

4) $z = 16\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ bo`lsa.

2. $z^5 = r(\cos\varphi + i\sin\varphi)$ tenglamaning barcha ildizlari uchun formula yozing.

3. Ikki hadli tenglamalarni yeching:

1) $z^3+i=0$ 2) $z^4=i$

3) $z^5+1=0$ 4) $z^4-32=0$

4. Kompleks tekislikda berilgan songa mos keluvchi nuqtani va vektorni yasang:

1) z=2-3i; 2) z=-3-2i; 3) z=3;

4) z=2i; 5) z=-1-3i; 6) z=5-4i;

7) z=(3-2i)(1-2i) 8) $z = \frac{4-i}{2+2i}$.

5. Kompleks sonni trigonometrik shaklda yozing.

1) z=1-i; 2) z=1+i; 3) z=-1-i;

4) z=1; 5) z=2; 6) z=-i

7) z=i; 8) $z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$; 9) $z = \sqrt{3} + i$

10) $z = \sqrt{2} - \sqrt{2}i$; 11) $z = 2i$; 12) $z = \sqrt{3} - \sqrt{3}i$;

13) $z = 2\cos\frac{5\pi}{4} - 2\sin\frac{5\pi}{4}$; 14) $z = -\cos\frac{\pi}{15} + i\sin\frac{\pi}{15}$.

6. z va w kompleks sonlarni trigonometrik shakldа yozing va zw hamda $\frac{z}{w}$ amallarni bajaring.

1) z=1+i; $w = \sqrt{3} - i$

2) z=2-2i; $w = 1 + \sqrt{3}i$

3) z=3+3i; w=i

4) z=2i; w=-4+4i.

7. a) z kompleks son berilgan. z^3, z^4, z^6 topilsin:

1) z=1-i; 2) z=i.

b) Quyidagi sonlarni algebraik ko`rinishda yozing.

1) $z = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$; 2) $z = 3(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$

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