SOLVING LINEAR EQUATIONS AND NONLINEAR EQUATION IN APPLICATION OF EXCEL

Matnazarov Ulug`bek Baxromovich
Associate professor of the Department of Mathematics and its Teaching Methodology, Candidate of Pedagogical Sciences
Tatarina Valentina Aleksandrovna
(TDPU named after Nizomi) 5110100-Bachelor of Mathematics and Informatics second specialis
Ten Viktoriya Anatolevna
(TDPU named after Nizomi) 5110100-Bachelor of Mathematics and Informatics second specialis
Deyneka Dmitriy Ivanovich
(TDPU named after Nizomi) 5110100-Bachelor of Mathematics and Informatics second specialis
Zairov Ravshan Baxtiyarovich
(TDPU named after Nizomi) 5110100-Bachelor of Mathematics and Informatics second specialis
Nasirov Batiy-xan Sultanovich
(TDPU named after Nizomi) 5110100-Bachelor of Mathematics and Informatics second specialis

Abstract: *Solutions to linear equations and nonlinear equation are not only the main teaching contents of the course Computer Method , but also widely applied in engineering. It's difficult for non-computer major students to program and understand abstract theoretical explanations of mathematics. During the teaching process, the author finds out that it's vivid, intuitive and understandable to use Excel to solve linear equations and nonlinear equation. Taking equations* $10X_1-X_2-2X_3=7.2$, $-X_1+10X_2-2X_3=8.3$, $-X_1-X_2+5X_3=4.2$ *and* $x^3+2x^2-16x-20=0$ *as instance, the author of this paper respectively explains the solution process of Jacobi iteration method, Seidel iteration method by Excel linear equations , and the solution process of Progressive scanning method , Newton iteration method, Interpolation method and Two-point chord cutting method by Excel nonlinear equation as well.*

Keywords: *Excel; linear equations; Nonlinear equation.*

INTRODUCTION

Solutions to linear equations and nonlinear equation are not only the main teaching contents of the course Computer Method , but also widely applied in engineering. The previous solution is mainly achieved by programming which is difficult for non-computer major students to some extent and the theoretical explanation of mathematics is also quite abstract. During the teaching process the author finds out it's vivid and understandable to use Excel Software to explain solving process. Excel Software can transfer boring theories into intuitive data, which helps students grasp solution method and process rapidly and meanwhile promotes students' interests in learning this course.

II.The Application of Excel in Solving Linear
Equations

Jacobi iteration method and Seidel iteration method are the basic algorithms to solve linear equations, and their solution processes are similar. The author takes equations $10X_1-X_2-2X_3=7.2$, $-X_1+10X_2-2X_3=8.3$ and $-X_1-X_2+5X_3=4.2$ as

instance to discuss the solution process of linear equations by Excel Jacobi iteration method and Seidel iteration method.

Original equations: equations' deformations:

$10X_1-X_2-2X_3=7.2$ $X_1=0.1X_2+0.2X_3+0.72$

$-X_1+10X_2-2X_3=8.3$ $X_2=0.1X_1+0.2X_3+0.83$

$-X_1-X_2+5X_3=4.2$ $X_3=0.2X_1+0.2X_2+0.84$

A. Jacobi iteration method to solve linear equations

Jacobi iteration method iterative format:

$X_1^{k+1}=0.1X_2^{k}+0.2X_3^{k}+0.72$

$X_2^{k+1}=0.1X_1^{k}+0.2X_3^{k}+0.83$

$X_3^{k+1}=0.2X_1^{k}+0.2X_2^{k}+0.84$

Start Excel Software and put x1, x2 and x3 in cell B1, C1 and D1. Choose a group of initials such as (0,0,0) at random, put 0 into cell B2, C2 and D2 respectively, and put the formula =0.1*C2+0.2*D2+0.72, =0.1*B2+0.2*D2+0.83,=0.2*B2+0.2*C2+0.84 in cell B3, C3 and D3,and drag fill handle to calculate something else. Six decimals are reserved in the results. We can find from the outcomes that the data are unchangeable until after the fourteenth iteration, that is to say we work out the answer. The answer is $X_1=1.1$, $X_2=1.2$, $X_3=1.3$. Iteration results shown in Table 1.

	X1	X2	X2
0	0	0	0
1	0.720000	0.830000	0.840000
2	0.971000	1.070000	1.150000
3	1.057000	1.157100	1.248200
……			
13	1.099999	1.199999	1.299999
14	1.100000	1.200000	1.300000
15	1.100000	1.200000	1.300000
……			

	X1	X2	X2
0	0	0	0
1	0.720000	0.902000	1.164400
2	1.043080	1.167188	1.282054
3	1.093130	1.195724	1.297771
4	1.099126	1.199467	1.299719
5	1.099890	1.199933	1.299965
6	1.099986	1.199992	1.299996
7	1.099998	1.199999	1.299999
8	1.100000	1.200000	1.300000
9	1.100000	1.200000	1.300000
......			

B. Seidel iteration method to solve linear equations

Seidel iteration method to solve linear equations

Seidel iteration method iterative format:

X1 k+1=0.1X2 k +0.2X3 k +0.72

X2 k+1= 0.1X1 k+1+0.2X3 k +0.83

X3 k+1=0.2X1 k+1+0.2X2 k+1+0.84

Start Excel Software and put x1, x2 and x3 in cell B1, C1 and D1. Choose a group of initials such as (0,0,0) at random, put 0 into cell B2, C2 and D2 respectively, and put the formula =0.1*C2+0.2*D2+0.72,=0.1*B3+0.2*D2+0.83 and =0.2*B3+0.2*C3+0.84 in cell B3, C3 and D3,and drag fill handle to calculate something else. Six decimals are reserved in the results. We can find from the outcomes that the data are unchangeable until after the eighth iteration, that is to say we work out the answer. The answer is X1=1.1,X2=1.2,X3=1.3 . Iteration results shown in Table 2.

The basic theory of Jacobi iteration method and Seidel

iteration method is similar. We only make use of the previous iteration value when we use Jacobi iteration method to solve problems, while we make full use of the updated iteration value when we use Seidel iteration method to solve problems. We can find that Seidel iteration method is more efficient than Jacobi iteration method to iterate through the iteration results.

III The Application of Excel in Solving Nonlinear Equation

Within a certain range, we have two steps to work out the solution to nonlinear equation. Step 1: work out the approximate interval of root. Step 2: make the root precise. The graphic method and progressive scanning method can work out the approximate interval of root. We usually use such methods as Newton iteration, interpolation and two-point chord cutting method to make the root precise. The author of this paper takes equations $x^3+2x^2-16x-20=0$ as instance to discuss various solutions to nonlinear equation by Excel.

A. Fix the approximate intervals of the root

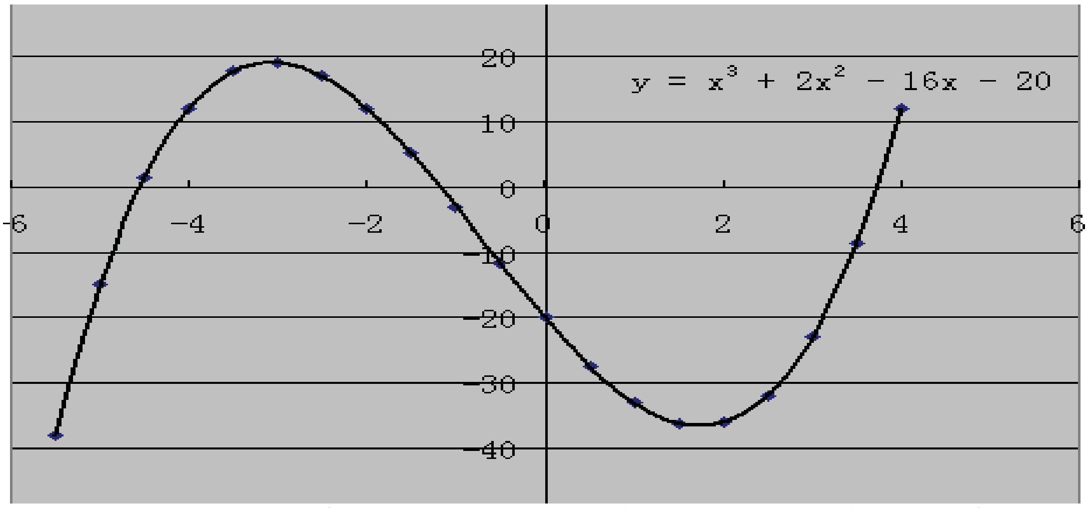
1) The scatter figure of XY works out the numbers and intervals of the root

Setting y= $x^3+2x^2-16x-20$, When x=-5,-4......4, 5, its corresponding value of Y is shown in Table 3.

TABLE 3

x	y	x	y
-5	-15	-4	12
-2	12	-3	19
-1	-3	0	-20
1	-33	2	-36
3	-23	4	12
5	75		

Fig 1: The XY scatter figure of the equation $x^3+2x^2-16x-20$ by Excel



We make the XY scatter figure by Excel according to Table 3, shown in fig.1.

The first derivative of given function is: $3x^2+4x-16$. When X less than -4 and more than 4, the first derivatives of given function are more than zero. The function increases monotonously. Therefore when X less than -4 and more than 4, there are no real roots. We can see the equation

$x^3+2x^2-10x-20=0$ has three real roots(between -5 and 4) from the diagram.

2) Progressive scanning method to determine the root of each interval

After using the graphic method determines the numbers and general range of the root, we can use progressive scanning method to determine the scope of each root gradually according to given steps. Taking the equation

$x^3+2x^2-16x-20=0$ as instance, the given step 0.2, we make use of progressive scanning method to determine the specific scope of the root (between interval -5 and 4).

Setting y= $x^3+2x^2-16x-20$, calculate the value of y respectively, when x=-5, -4.8......3.8, 4. Part of the data shown in Table 4.

TABLE 4

x	y	result
-4.8	-7.712	
-4.6	-1.416	negative
-4.4	3.936	positive

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-1.2 -1	0.352 -3	positive negative
ĂĂ		
3.6 3.8	-5.024 2.952	Negative positive
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According to Table 4, when x=-5, -4.8……-4.6, the corresponding values of y are all negative. When x=-4.4, the corresponding value of y is positive. All these show that between the interval-4.6 and -4.4, the equation has real root, and the same as between the interval -1.2 and -1, the interval

3.6 and 3.8.

B. Making the root precise

1) Newton iteration method

The author takes advantage of Newton iteration method to calculate the value of the equation $x^3+2x^2-16x-20=0$ within the range of -4.6 and -4.4, which is used as an example to explain the solution process of Excel by Newton iteration method. Newton iteration method is the most basic algorithm of solution to nonlinear equation by the formula

$x_{n+1}=x_n-f(x_n)/f'(x_n)$. When using Newton iteration method, we are required to pay attention to the choice of initials, and the choice principle is that the function value of the initials and the second derivatives are the same sign.

We are required to determine the initial before using Excel to solve the question.

$f(x)=x^3+2x^2-16x-20$ $f'(x)=3x^2+4x-16$

f''(x)= 6x+4 ‹0∧-4.6'-4.5∨

f∧-4.5∨›0 f∧-4.6∨‹0

Because the function value of the initials and the second derivatives are the same sign, the initial is chosen as -4.6.

Put -4.6 into cell B1 and formula

=B1-(B1^3+2*B1^2-16*B1-20)/(3*B1^2+4*B1-16) into cell B2, drag filling handle B2 downward, and calculate the others. The results shown in Table 5.

TABLE 5

0	-4.6
1	-4.55130674
2	-4.550309432
3	-4.550309017
4	-4.550309017
5	-4.550309017
ĂĂ	

TABLE 6

0	-1.2
1	-1.178997613
2	-1.178688829
3	-1.178684367

4	-1.178684302
ĂĂ	

TABLE 7

0	3.6
1	3.8
2	3.725977934
3	3.728924987
4	3.728993385
5	3.728993318
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Nine decimals are reserved in the results. Calculate to the fourth line, the data unchangeable. Therefore, making use of Newton iteration method to solve the root of equation

$x^3+2x^2-16x-20=0$ within the range of -4.6 and -4.5, the root is:

-4.550309017.

2) Interpolation

The author makes use of interpolation to calculate the root of equation $x^3+2x^2-16x-20=0$ within the range of -1.2 and -1, which is used as an example to explain the solution process of Excel by the method of interpolation.

The principle is:

When f(x) first derivative and f(x) second derivative are the same sign, getting $x_0=a$, Iterative formula: $x_{n+1}=x_n-x_n-b*f(x_n)/f(x_n)-f(b)$;

When f(x) first derivative and f(x) second derivative are the different sign, getting $x_0=b$, Iterative formula: $x_{n+1}=x_n-X_n-a*f(x_n)/f(x_n)-f(a)$;

$f(x)=x^3+2x^2-16x-20$, $f'(x)=3x^2+4x-16$, $f''(x)=6x+4$, The values of f(x) first derivative and f(x) second derivative within the range of -1.2 and -1 are all less than zero, that is to say, they have the same sign. Therefore $x_0=-1.2$, Newton iteration formula: $x_{n+1}=x_n-x_n-b*f(x_n)/f(x_n)-f(b)$ b=-1 f(b)=-3, putting -1.2 into cell B1, putting the formula

=B1-(B1+1)*(B1^3+2*B1^2-16*B1-20)/((B1^3+2*B1^2-16*

B1-20)+3) into cell B2, the results shown in Table 6.

Nine decimals are reserved in the results. When we calculate to the fourth line, the results keep unchangeable. Thus, the real roots of equation $x^3+2x^2-16x-20=0$ within the range of -1.2 and -1 by the method of interpolation are:

-1.178684302

3) Two-point chord cutting method

The iteration formula of two-point chord cutting method is:

$x_{n+1}=x_n-(x_n-x_{n-1})*f(x_n)/(f(x_n)-f(x_{n-1}))$, making use of two points. The author calculates the roots of equation $x^3+2x^2-16x-20=0$ within the range of 3.6 and 3.8 by the

two-point chord cutting method, which is used as an instance to explain the solution process of Excel by the method of two-point chord cutting.

Start Excel Software, put 3.6 in cell B1, 3.8 in cell B2, formula

=B2-(B2-B1)*(B2^3+2*B2^2-16*B2-20)/((B2^3+2*B2^2-16

*B2-20)-(B1^3+2*B1^2-16*B1-20)) in cell B3, and drag filling handle B2 downward, and calculate the others. The results shown in Table 7.

Nine decimals are reserved in the results. The real roots of equation $x^3+2x^2-16x-20=0$ within the range of 3.6 and 3.8 by the two-point chord cutting method are : 3.728993318

IV.Conclusion

Excel Software is everywhere, which can not only be used to help people to solve problems in daily life, but various calculating problems usually encountered at work. During the teaching process of Computational (Method) course, the author has sensed profoundly that solving the questions with the help of Excel in class can not only change boring theories into simple calculation, but also arouse the students' interest in learning, and then help students understand the theory and solution process of algorithm deeply. Excel is not only applied to the solution of linear equations and nonlinear equation, but applied to the other problems such as recursive algorithm etc. during the teaching process, and the author has gained excellent teaching effectiveness.

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