РЕШЕНИЕ КРЫВЫХ ЗАДАЧИ УРАВНЕНИЙ ГРАНИЦЫ ДЛЯ ТЕПЛОПЕРЕДАЧИ КРАЕВАЯ ПОЛЕВАЯ ЗАДАЧА В ДВИЖУЩЕМСЯ ПОЛЕ

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Аннотация: *В данной диссертации предлагается поставить краевую задачу для уравнения теплообмена в движущейся сфере (нецилиндрических сферах) и решить ее с помощью функции Грина.*

Ключевые слова: *Кусочно-гладкая поверхность, непрерывно дифференцируемая функция, функции Грина, задачи уравнений границы, температурная функция, нецилиндрическая область.*

Annotation: *In this dissertation, it is proposed to set a boundary value problem for the heat transfer equation in a moving sphere (non-cylindrical spheres) and solve it using the Green's function.*

Key words: *Piecewise-smooth surface, continuously differentiable function, Green's functions, problems of boundary equations, temperature function, non-cylindrical domain.*

Annotatsiya: *Ushbu tezisda issiqlik o'tkazuvchanlik tenglamasi uchun chegaralari harakatlanuvchi sohada (silindrsimon bo'lmagan sohalar) chegaraviy masalaning qo'yilishi va uning yechimi Grin funksiyasidan foydalanib yechish taklif etilgan.*

Kalit so'zlar: *Bo'lakli silliq sirt, uzluksiz differensiallanuvchi funksiya, Grin funksiyalari, chegaraviy masalalar, issiqlik funksiyasi, silindrsimon bo'lmagan sohalar.*

Chegaraviy masalaning qo'yilishi. Bizga *(n+1)*o'lchovli fazoda Ω_i - silindrsimon bo'lmagan soha berilgan bo'lsin. Shu sohaning xarakteristik tekisligida yotuvchi $t = const \geq t_0 > 0 : D_t \left(D_t \in \mathbb{R}^n \right)$ qavariq sohadan $M\left(x_1, x_2, ..., x_n\right)$ nuqtani olaylik. S_t-esa vaqtga bog'liq ravishda D_t sohani $\left(t \geq 0\right)$ chegaralovchi bo'lakli silliq sirt bo'lsin, *n*-esa S_t uchun tashqi normal o'lchovi bo'lsin. Shunday qilib $\Omega_t = \left\{ M \in \overline{D_t} = D_t \cup S_t, t \geq 0 \right\}$.

$T(M,t)$ -issiqlik funksiyasi quyidagi shartlarni qanoatlantirsin:

$$\frac{\partial T}{\partial t} = a\Delta T(M,t) + f(M,t), \quad M \in D_t, \quad t > 0; \qquad (1)$$

$$T(M,t)\big|_{t=0} = \Phi_0(M), \quad M \in \bar{D}_{t=0}; \qquad (2)$$

$$\beta_1 \frac{\partial T(M,t)}{\partial n} + \beta_2 T(M,t) = \varphi(M,t) \quad M \in S_t, \ t \ge 0 \qquad (3)$$

(1)–(3) shartlardagi chegaraviy funksiyalar quyidagi yechimlar sinfiga tegishli

$f(M,t) \in C^0(\bar{\Omega}_t)$; $\Phi_0(M) \in C^1(\bar{\Omega}_t)$;

$\varphi(M,t) \in C^0(S_t t \ge 0)$; $\beta_1^2 + \beta_2^2 > 0$.

Kutilayotgan yechim esa:

$T(M,t) \in C^2(\Omega_t) \cap C^0(\bar{\Omega}_t)$,

$grad_M T(M,t) \in C^0(\bar{\Omega}_t)$.

Silindrsimon sohalarda bo‘lgani kabi harakatlanuvchi chegaralari bo‘lgan sohalarda (silindrsimon bo‘lmagan sohalar) ham birinchi $(\beta_1 = 0)$, ikkinchi $(\beta_2 = 0)$ yoki uchinchi $(\beta_i > 0, i = 1,2)$ chegaraviy masalalarni ko‘rib chiqamiz. Biroq chegaraviy masalada ko‘rsatilgan ekvivalentlik munosabatlar har doim ham saqlanib qolmaydi. Xususan, $t > 0$da harakatlanuvchi chegaraning issiqlik tarqalish holati $y(t)$ funksiya ham, bu yerda $-y(t)$ uzluksiz differentsiallanuvchi har qanday tartibdagi chekli hosilalari bo‘lgan funksiya quyidagi ko‘rinishga ega [1]:

$$\left[\frac{\partial T(x,t)}{\partial x} + \frac{v(t)}{a} T(x,t)\right]_{x=y(t)} = 0, \qquad t > 0, \qquad (4)$$

va harakat tezligi $v(t) = \frac{d\left(y(t)\right)}{dt} = 0 \ \left(y(t) = const\right)$. (4) ifoda klassik issiqlik tarqalish tenglamasi uchun Furye qonunidan kelib chiqadigan skalyar forma hisoblanadi [2]. Keyinchalik bu, sohaning o‘ziga xos xususiyatlaridan kelib chiqib harakatlanuvchi chegara bo‘yicha Grin funksiyalari uchun ham chegaraviy masalalarni shakllantirishga sabab bo‘ldi. Bu yerda, (2) va (3)- chegaraviy masala uchun Grin funksiyalarini topishga alohida e’tibor berishimiz lozim.

Bizga chegarasi bo‘lakli silliq S (S ning tashqi o‘lchovi $n$ga teng) sirt qoplangan D qavariq sohada chegaralangan yoki qisman chegaralangan $M(x,y,z)$ funksiya berilgan bo‘lsin. Hamda t vaqt bo‘yicha D sohaning xarakteristik tekisligi $(t = const \ge t_0 \ge 0)$ $D_t$ga qism bo‘lgan Ω_t- chegaralari harakatlanuvchi soha (silindrsimon bo‘lmagan soha) berilgan bo‘lsin:

$\Omega_t = \{ y_1(t) < x \le y_2(t), t > 0 \}$, bu yerda $y_i(t)$- uzluksiz differensiallanuvchi funksiyalar.

1-teorema. $T(x,t) \in \Omega_t$ funksiya quyidagi

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} + f(x,t) \qquad (5)$$

tenglamaning boshlang'ich

$$T(x,0) = \Phi_0(x), y_1(0) \le x \le y_2(0) \qquad y_i(0) \ge 0, i = 1,2 \qquad (6)$$

va chegaraviy

$$\left[\beta_{i1}\frac{\partial T(x,t)}{\partial x} + \beta_{i2}T(x,t) \right]_{x=y_i(\tau)} = \beta_{i3}\varphi_i(t), \quad t \ge 0, i = 1,2 \qquad (7)$$

shartlani qanoatlantiradi.

Bu yerda (7) chegaraviy masalada quyidagi hollardan biri bo'lishi mumkin:

a) Yoki birinchi turdagi $(\beta_{i1} = 0, \beta_{i2} = \beta_{i3} = 1)$;

b) Yoki ikkinchi turdagi $(\beta_{i2} = 0, \beta_{i1} = \beta_{i3} = 1)$;

c) Yoki uchinchi turdagi $(\beta_{i1} = 1, \beta_{i2} = \beta_{i3} = (-1)^i h_i$, bu yerda h_i-nisbiy issiqlik tarqalish koeffitsenti).

Agar $G(x,t,x',\tau)$-Grin funksiyasi quyidagi chegaraviy shartlarga ajratilib qaralsa,yani:

a) birinchi chegaraviy masala

$$G(x,t,x',\tau)\big|_{x'=y_i(\tau)} = 0, \qquad \tau < t, i = 1,2; \qquad (8)$$

b) ikkinchi chegaraviy masala

$$\left(\frac{\partial G}{\partial x'} - \frac{1}{a}\frac{\partial y_i}{\partial \tau}\right)\Bigg|_{x'=y_i(\tau)} = 0, \qquad \tau = t, i = 1,2; \qquad (9)$$

c) uchinchi chegaraviy masala

$$\left(\frac{\partial G}{\partial x'} + (-1)^j \left[h_i + (-1)^{i-1}\frac{1}{a}\frac{\partial y_i}{\partial \tau} \right] G\right)\Bigg|_{x'=y_i(\tau)} = 0 \qquad (10)$$

bo'lsa, u holda $T(x,t)$ yechim ham quyidagi hollarga ajratilib topiladi:

a) birinchi chegaraviy masala uchun:

$$T(x,t) = \int_{y_1(0)}^{y_2(0)} T(x',\tau)G(x,t,x',\tau)dx' + a\int_0^t \left[T(x',\tau)\frac{\partial G}{\partial x'} \right]\Bigg|_{x'=y_1(\tau)} d\tau -$$

$$a\int_0^t \left[T(x',\tau)\frac{\partial G}{\partial x'} \right]\Bigg|_{x'=y_2(\tau)} d\tau + \int_0^t \int_{y_1(\tau)}^{y_2(\tau)} f(x',\tau)G(x,t,x',\tau)d\tau dx'$$

b) ikkinchi chegaraviy masala uchun:

$$T(x,t)=\int_{y_1(0)}^{y_2(0)}\left[T(x',\tau)G(x,t,x',\tau)\right]\Big|_{\tau=0}dx'-a\int_0^t\left[\frac{\partial T(x',\tau)}{\partial x'}G(x,t,x',\tau)\right]\Big|_{x'=y_1(\tau)}d\tau+$$

$$a\int_0^t\left[\frac{\partial T(x',\tau)}{\partial x'}G(x,t,x',\tau)\right]\Big|_{x'=y_2(\tau)}d\tau+\int_0^t\int_{y_1(\tau)}^{y_2(\tau)}f(x',\tau)G(x,t,x',\tau)d\tau dx'$$

c) uchinchi chegaraviy masala uchun:

$$T(x,t)=\int_{y_1(0)}^{y_2(0)}\left[T(x',\tau)G(x,t,x',\tau)\right]\Big|_{\tau=0}dx'-a\int_0^t\left[\left(\frac{\partial T(x',\tau)}{\partial x'}-h_1T(x',\tau)\right)G(x,t,x',\tau)\right]\Big|_{x'=y_1(\tau)}d\tau+$$

$$a\int_0^t\left[\frac{\partial T(x',\tau)}{\partial x'}+h_2T(x',\tau)G(x,t,x',\tau)\right]\Big|_{x'=y_2(\tau)}d\tau+\int_0^t\int_{y_1(\tau)}^{y_2(\tau)}f(x',\tau)G(x,t,x',\tau)d\tau dx'$$

Endi (3.2.4) koʻrinishdagi $G(x,t,x',\tau)$-Grin funksiyasini koʻrib chiqamiz.

Quyidagi $\bar{G}(x,t,x',\tau)$ funksiyani qaraylik.

Bu funksiya quyidagi boshlangʻich-chegaraviy shartlarni qanoatlantirsin:

$$\frac{\partial\bar{G}}{\partial t}=a\frac{\partial^2\bar{G}}{\partial x^2},\quad y_1(t)<x<y_2(t),t>\tau \qquad (11)$$

$$\bar{G}(x,t,x',\tau)\Big|_{t=\tau}=\delta(x-x')\qquad y_1(\tau)<x'<y_2(\tau) \qquad (12)$$

$$\left[\beta_{i1}\frac{\partial\bar{G}}{\partial x}+\beta_{i2}\bar{G}\right]\Big|_{x=y_i(t)}=0,\qquad t>\tau \qquad (13)$$

bu yerda $\beta_{i1}=0$,agar birinchi chegaraviy masala uchun, $\beta_{i2}=0$ ikkinchi chegaraviy masala va $\beta_{i1}=1,\beta_{i2}=\beta_{i3}=(-1)^i h_i$ uchinchi chegaraviy masala uchun.

2-teorema. (5)-(7) shartlarni qanoatlantiruvchi $\bar{G}(x,t,x',\tau)$ Grin funksiya mavjud va yagona boʻladi, yaʼni $\bar{G}(x,t,x',\tau)\equiv G(x,t,x',\tau)$ tenglik oʻrinli boʻladi.

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