LOCAL INNER DERIVATIONS ON FOUR-DIMENSIONAL LIE ALGEBRAS

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Annotation: *A number of difficulties arise in studying the local internal differentiation of small Lie algebras. This article therefore considers local inner differentiations of the four-dimensional Lie algebras. At the same time a general view of differentials of Lie algebras has been found.*

Key words: *Lie algebra, derivation, inner derivation, local derivation, local inner derivation*.

Definition 1. An algebra L over field F is called a *Liera* if its multiplication satisfies the identities:

1) $[x,x]=0$,
2) $[x,[y,z]]+[y,[z,x]]+[z,[x,y]]=0$,

for all x,y,z in L.

The product $[x,y]$ is called the bracket of x and y. Identity 2) is called the Jacobi identity.

Let L be a finite-dimensional Lie algebra. For Lie algebra L we consider the following central and derived series:

$$L^1=L,\ L^i=\left[L^{i-1},L\right],\ i\geq 1,$$

$$L^{[1]}=L,\ L^{[k]}=\left[L^{[k-1]},L^{[k-1]}\right],\ k\geq 1.$$

A Lie algebra L is *nilpotent (solvable)* if there exists $m\geq 1$ such that $L^m=0$ ($L^{[m]}=0$).

Definition 2. A *derivation* on a Lie algebra L is a linear map $D:L\to L$ which satisfies the Leibniz rule:

$$D([x,y])=[D(x),y]+[x,D(y)]$$

for any $x,y\in L$. The set of all derivations of a Lie algebra denoted by $Der(L)$. Let a be an element of a Lie algebra L. Consider the operator of $ad_a:L\to L$ defined by $ad_a(x)=[x,a]$. This operator is a derivation and called *inner derivation.* The set of all inner derivations of a Lie algebra denoted by $InDer(L)$.

Definition 3. A linear operator Δ is called a *local derivation* if for any $x\in L$, there exists a derivation $D_x:L\to L$ (depending on x) such that $\Delta(x)=D_x(x)$.

Definition 4. A linear operator Δ is called a *local inner derivation* if for any $x\in L$, there exists a inner derivation $ad_x:L\to L$ (depending on x) such that $\Delta(x)=ad_x(x)$.

We present the following theorem which gives a classification of arbitrary four-dimensional Lie algebras.

Theorem 1. [1]. An arbitrary four-dimensional Lie algebra is isomorphic to one of the following algebras:

L_0 : abelian; $L_1:[e_1,e_2]=e_3$; $L_2:[e_1,e_2]=e_1$; $L_3:[e_1,e_2]=e_2,[e_1,e_3]=e_2+e_3$;

$L_4:[e_1,e_2]=e_2,[e_1,e_3]=\lambda e_3,\lambda\in C^*,|\lambda|\leq 1$; $L_5:[e_1,e_2]=e_1,[e_3,e_4]=e_3$;

$L_6:[e_1,e_2]=e_3,[e_1,e_3]=-2e_1,[e_2,e_3]=2e_2$; $L_7:[e_1,e_2]=e_3,[e_1,e_3]=e_4$;

$L_8:[e_1,e_2]=e_2,[e_1,e_3]=e_3,[e_1,e_4]=\alpha e_4,\alpha\in C^*$;

$L_9:[e_1,e_2]=e_3,[e_1,e_3]=e_4,[e_1,e_4]=\alpha e_2-\beta e_3-e_4,\alpha\in C^*,\beta\in C$;

$L_{10}:[e_1,e_2]=e_3,[e_1,e_3]=e_4,[e_1,e_4]=\alpha(e_2+e_3),\alpha\in C^*$;

$L_{11}:[e_1,e_2]=e_3,[e_1,e_3]=e_4,[e_1,e_4]=e_2$; $L_{12}:[e_1,e_2]=\frac{1}{3}e_2+e_3,[e_1,e_3]=\frac{1}{3}e_3,[e_1,e_4]=\frac{1}{3}e_4$;

$L_{13}:[e_1,e_2]=e_2,[e_1,e_3]=e_3,[e_1,e_4]=2e_4,[e_2,e_3]=e_4$;

$L_{14}:[e_1,e_2]=e_3,[e_1,e_3]=e_2,[e_2,e_3]=e_4$;

$L_{15}:[e_1,e_2]=e_3,[e_1,e_3]=-\alpha e_2+e_3,[e_1,e_4]=e_4,[e_2,e_3]=e_4,\alpha\in C$;

Theorem 1.[2-3]. Any local inner derivation on the algebras L_1 – L_{12}, L_{14} is an inner derivation, and on the algebras L_{13} and L_{15} there exists a local inner derivation which is not an inner derivation.

This means that the operator Δ is an inner derivation.

In four-dimensional Lie algebras L_1-L_{12} and L_{14}, an arbitrary local inner derivation is an inner derivation.

The algebras L_{13} and L_{15} admit a local inner derivation that is not an inner derivation.

Algebra L_{13}. Consider an operator

$$\Delta=(a_{42}x_2+a_{43}x_3+a_{44}x_4)e_4.$$

This operator is a local inner derivation.

For function

$$\varphi(x)=\begin{cases}\frac{1}{x_2}(a_{42}x_2+a_{43}x_3+a_{44}x_4)e_3, & x_1=0,\ x_2\neq 0,\\ -\frac{a_{44}}{2}e_1, & x_1=x_2=x_3=0,\\ -\frac{1}{x_3}(a_{43}x_3+a_{44}x_4)e_2, & x_1=x_2=0,\ x_3\neq 0,\\ \frac{1}{2x_1}(a_{42}x_2+a_{43}x_3+a_{44}x_4)e_4, & x_1\neq 0.\end{cases}$$

$$\Delta(x)=[x,\varphi(x)].$$

Algebra L_{15}. As in the case of the algebra L_{13}, it is shown that

Consider an operator

$$\Delta=a_{21}x_1e_2+a_{31}x_1e_3.$$

This operator is a local intrinsic derivation.[4-6].

For function

$$\phi(x)=\begin{cases}0,\ x_1=0,\\ (a_{31}+\dfrac{a_{21}}{\alpha})e_2-\dfrac{a_{21}}{\alpha}e_3+\dfrac{(\alpha a_{31}+a_{21})x_3+a_{21}x_2}{\alpha x_1},\ \ x_1\neq 0.\end{cases}$$

$$\Delta(x)=[x,\phi(x)].$$

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