ODDIY XOS QIYMATLARGA MOS KELUVCHI MOSLANGAN MANBALI MODIFITSIRLANGAN KORTEVEG-DE FRIZ (MKDF) TENGLAMASINI INTEGRALLASH

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Annotatsiya: *ushbu maqolada modifitsirlangan Korteveg-de Friz tenglamasini oddiy va xos qiymatlarga mos keluvchi moslangan manba bilan integrallash keltirilgan*.

Kalit so'zlar: *xos qiymat, xos vektor funksiya, "tez kamayuvchi" funksiyalar sinfi, Loran qatori, xos funksiya, integral tenglama,*

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Quyidagi tenglamalar sistemasini ko'rib chiqamiz.

$$\begin{cases} u_t + 6u^2 u_x + u_{xxx} = \sum_{k=1}^{2N} (f_{k1} g_{k1} - f_{k2} g_{k2}), \\ L(t) f_k = \xi_k f_k, \quad L(t) g_k = \xi_k g_k, \quad k = 1,2,\ldots,2N, \end{cases} \tag{1}$$

Quyidagi boshlang'ich shart berilgan bo'lsin

$$u(x,0) = u_0(x), \quad x \in R^1, \tag{2}$$

Bunda $U_0(x) \quad (-\infty < x < +\infty)$ quyidagi xossalarga ega:

1. $$\int_{-\infty}^{\infty} (1 + |x|) \, |u_0(x)| dx < \infty; \tag{3}$$

2. $L(0)$ operator 2N larda quyidagi qiymatlarga ega: $\xi_1(0), \xi_2(0), \ldots, \xi_{2N}(0)$ ko'rib chiqilayotgan $Lg_k = \xi_k g_k$ masalada xos qiymatga mos keladigan $L(t)$ operatorning xos vektor funksiyasi $f_k = (f_{k1}, f_{k2})^T$ va $g_k = (g_{k1}, g_{k2})^T$ tenglamaning yechimi ξ_k, buning uchun

$$W\{f_k, g_k\} \equiv f_{k1} g_{k2} - f_{k2} g_{k1} = \omega_k(t) \neq 0, \quad k = \overline{1,2N}, \tag{4}$$

bu yerda $\omega_k(t)$ dastlab shartlarni qanoatlantiruvchi uzluksiz funksiyalar berilgan.

$$\omega_n(t) = -\omega_k(t) \qquad agar \qquad \xi_n = -\xi_k,$$
$$\operatorname{Re}\left\{\int_0^t \omega_k(\tau)d\tau\right\} > -\operatorname{Im}\{\xi_k(0)\}, \quad k = \overline{1,N} \tag{5}$$

t ning barcha xos bo‘lmagan qiymatlari uchun (1) ning o‘ng tomonida ishtirok etayotgan yig‘indiga birinchi navbatda $\operatorname{Im}\xi_k > 0, \;\; k = \overline{1,N}$ deb shartlar beramiz .

“Tez kamayuvchi” funksiyalar sinfidan U(x) funksiya $u(x,t), f_k, g_k, \;\; k = \overline{1,2N},$ bo‘lsin. Ushbu bo‘limning asosiy maqsadi $L(t)$ operator uchun teskari tarqalish usuli doirasida (1) - (5) muammoning yechimlari uchun tasvirlarni olishdir.

$h_n(x)$ funksiya uchun quyida keltirilgan tenglikka ko’ra

$$h_n(x) = \frac{\frac{d}{d\xi}(\varphi - C_n\psi)\Big|_{\xi=\xi_n}}{\dot{a}(\xi_n)}, \quad n = \overline{1,N},$$

$$h_n(x) = \frac{\beta_n}{\dot{a}(\xi_n)}\varphi(x,\xi_n) + \alpha_n g_n, \;\; n = \overline{1,N}. \tag{7}$$

(7) tenglikka keladi. Bu yerda , $\dot{a}(\xi_n)$ noldan farq qiladi.

$$\begin{cases} v_{1x} + i\xi v_1 = u(x)v_2, \\ v_{2x} - i\xi v_2 = -u(x)v_1, \end{cases}$$

tenglamalar sistemasidagi u(x) potensial tenglamaning yechimi bo'lsin

$$u_t + 6u^2u_x + u_{xxx} = G(x,t)$$

Bu yerda G $x \to \pm\infty$da nolga aylanadi.

Lemma 1.1. Agar vektor funksiyani bajarsa va $LY = \xi Y$ va $LZ = \eta Z$ tenglamalarning yechimi bo‘lsa, unda ularning komponentlari tengliklarni qanoatlantiradi.

$$\frac{d}{dx}(y_1z_1 + y_2z_2) = -i(\xi + \eta)(y_1z_1 - y_2z_2),$$
$$\frac{d}{dx}(y_1z_2 - y_2z_1) = -i(\xi - \eta)(y_1z_2 + y_2z_1).$$

Ushbu lemmaning haqiqiyligi to‘g‘ridan-to‘g‘ri tekshirish bilan isbotlanadi.

Lemma 1.2.

Agar $y(x,\xi) = \begin{pmatrix} y_1(x,\xi) \\ y_2(x,\xi) \end{pmatrix}$ vektor funksiya

$$\begin{cases} v_{1x} + i\xi v_1 = u(x)v_2, \\ v_{2x} - i\xi v_2 = -u(x)v_1, \end{cases} \quad x \in R^1 \qquad (8)$$

sistemaning $\xi = u + iv$ songa mos keluvchi yechimi bo'lsa, u holda (8) sistemaning $\xi = u - iv$ songa mos keluvchi yechimi $\bar{y}(x,\xi) = \begin{pmatrix} \bar{y}_2(x,\bar{\xi}) \\ -\bar{y}_1(x,\bar{\xi}) \end{pmatrix}$ bo'ladi.

Lemma 1.2 va birinchi (5) shartlarga muvofiq, (1) tenglamaning o'ng tomoni

$$G(x,t) = 2\sum_{\substack{k=1, \\ \operatorname{Im}\xi_k>0}}^{N}(f_{k1}g_{k1} - f_{k2}g_{k2}).$$

shaklda qayta yozilishi mumkin.

Riman-Lebeg lemmasidan kelib chiqqan holda, manfiy bo'lmagan t uchun $G(x,t) = o(1)$ agar $|x| \to \infty$ ekanligini ko'rish oson.

$\varphi(x,\xi_k) = d_k\psi(x,\xi_k)$ natijalarini tenglamalar tizimiga (1) qo'llaymiz va sochilish nazariyasining evolyutsiyasini hisoblaymiz.

Quyidagi asimptotikalar $x \to -\infty$ uchun amal qiladi.

$$\varphi \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x}, \ \psi \sim \begin{pmatrix} \bar{b}e^{-i\xi x} \\ ae^{i\xi x} \end{pmatrix}, \ h_k \sim -C_k \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi_k x},$$

$$g_k \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \omega_k(t) e^{i\xi_k x}, \ \psi_k \sim \frac{1}{C_k}\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi_k x};$$

va $x \to \infty$ uchun

$$\phi \sim \begin{pmatrix} ae^{-i\xi x} \\ be^{i\xi x} \end{pmatrix}, \psi \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x}, \ h_k \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi_k x},$$

$$\sim -\begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{\omega_k(t)}{c_k} e^{-i\xi_k x}, \ \phi_k \sim C_k \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi_k x}.$$

Birinchidan $\int_{-\infty}^{+\infty} G(\phi_1^2 + \phi_2^2)\, dx$. ni hisoblasak. Lemma 1.1 ga binoan bizda quyidagi tenglik mavjud

$$(f_{k1}g_{k1} - f_{k2}g_{k2})(\phi_1^2 + \phi_2^2)$$
$$= f_{k1}g_{k1}\phi_1^2 + f_{k1}g_{k1}\phi_2^2 - f_{k2}g_{k2}\phi_1^2 - f_{k2}g_{k2}\phi_2^2 =$$
$$= \frac{1}{2}[(f_{k1}\phi_1 - f_{k2}\phi_2)(g_{k1}\phi_1 + g_{k2}\phi_2) + (f_{k1}\phi_1 + f_{k2}\phi_2)(g_{k1}\phi_1 - g_{k2}\phi_2)] +$$

$$+\frac{1}{2}[(f_{k1}\phi_2 - f_{k2}\phi_1)(g_{k2}\phi_1 + g_{k1}\phi_2) \qquad (9)$$
$$-(f_{k1}\phi_2 + f_{k2}\phi_1)(g_{k2}\phi_1 - g_{k1}\phi_2)] =$$

$$= \frac{1}{-2i(\xi+\xi_k)}\frac{d}{dx}[(f_{k1}\phi_1 + f_{k2}\phi_2)(g_{k1}\phi_1 + g_{k2}\phi_2)] - \quad \left(-\frac{1}{-2i(\xi-\xi_k)}\frac{d}{dx}[(\phi_1 f_{k2} - \phi_2 f_{k1})(\phi_1 g_{k2} - \phi_2 g_{k1})]\right.$$

Biz (9) tenglikni x boyicha -∞ dan +∞ gacha integrallasak

$$\frac{1}{-2i(\xi+\xi_k)}\int_{-\infty}^{\infty}\frac{d}{dx}[(f_{k1}\varphi_1 + f_{k2}\varphi_2)(g_{k1}\varphi_1 + g_{k2}\varphi_2)]dx =$$

$$= \frac{1}{-2i(\xi+\xi_k)}\lim_{R\to\infty} C_k b e^{i(\xi+\xi_k)R}\left(-\frac{\omega_k(t)}{C_k} a e^{-i(\xi+\xi_k)R}\right) = \frac{ab\omega_k(t)}{2i(\xi+\xi_k)}$$

Yuqoridagilarga ko‘ra

$$\int_{-\infty}^{\infty} G(\phi_1^2 + \phi_2^2)dx = -\sum_{k=1}^{N} iab\omega_k(t)\left(\frac{1}{\xi+\xi_k} + \frac{1}{\xi-\xi_k}\right). \qquad (10)$$

Shunday qilib

$$\frac{dr^+}{dt} = \left[8i\xi^3 + \sum_{k=1}^{N} i\omega_k(t)\left(\frac{1}{\xi+\xi_k} + \frac{1}{\xi-\xi_k}\right)\right] r^+.$$

Lemma 1.1 ga ko‘ra $\xi_k \neq \xi_n$

$$(f_{k1}g_{k1} - f_{k2}g_{k2})(h_{n1}\psi_{n1} + h_{n2}\psi_{n2}) =$$
$$= f_{k1}g_{k1}h_{n1}\psi_{n1} + f_{k1}g_{k1}h_{n2}\psi_{n2} - f_{k2}g_{k2}h_{n1}\psi_{n1} - f_{k2}g_{k2}h_{n2}\psi_{n2} =$$
$$= \frac{1}{-2i(\xi_n+\xi_k)}\frac{d}{dx}[(h_{n1}f_{k1} + h_{n2}f_{k2})(\psi_{n1}g_{k1} + \psi_{n2}g_{k1})] -$$
$$-\frac{1}{-2i(\xi_n-\xi_k)}\frac{d}{dx}[(h_{n1}f_{k2} - h_{n2}f_{k1})(\psi_{n1}g_{k2} - \psi_{k2}g_{k1})]. \qquad (11)$$

Biz tenglikni x boyicha -∞ dan +∞ gacha integrallasak

$$\frac{1}{-2i(\xi_n+\xi_k)}\int_{-\infty}^{\infty}\frac{d}{dx}[(h_{n1}f_{k1} + h_{n2}f_{k2})(\psi_{n1}g_{k1} + \psi_{n2}g_{k2})]dx -$$
$$-\frac{1}{-2i(\xi_n-\xi_k)}\int_{-\infty}^{\infty}\frac{d}{dx}[(h_{n1}f_{k2} - h_{n2}f_{k1})(\psi_{n1}g_{k2} - \psi_{n2}g_{k1})]dx =$$

$$=\frac{1}{-2i(\xi_n+\xi_k)}\lim_{R\to\infty}[(h_{n1}f_{k1}+h_{n2}f_{k2})(\psi_{n1}g_{k1}+\psi_{n2}g_{k2})]\Big|_{-R}^{R}+$$

$$+\frac{1}{2i(\xi_n-\xi_k)}\lim_{R\to\infty}[(h_{n1}f_{k2}-h_{n2}f_{k1})(\psi_{n1}g_{k2}+\psi_{n2}g_{k1})]\Big|_{-R}^{R}=$$

$$=\frac{1}{2i(\xi_n-\xi_k)}\lim_{R\to\infty}\left[C_k e^{i(-\xi_n+\xi_k)R}\frac{\omega_k(t)}{C_k}e^{i(\xi_n-\xi_k)R}-C_n e^{i(\xi_n-\xi_k)R}\frac{\omega_k(t)}{C_n}e^{-i(\xi_n-\xi_k)R}\right]=0.$$

Shunday qilib , $\xi_k\neq\xi_n$

$$\int_{-\infty}^{\infty}(f_{k1}g_{k1}-f_{k2}g_{k2})(h_{n1}\psi_{n1}+h_{n2}\psi_{n2})dx=0$$

Agar $\xi_k=\xi_n$ bo'lsa

$$(f_{n1}g_{n1}-f_{n2}g_{n2})(h_{n1}\psi_{n1}+h_{n2}\psi_{n2})=-\frac{1}{4i\xi_n}\frac{d}{dx}[(h_{n1}g_{n1}+h_{n2}g_{n2})(f_{n1}\psi_{n1}+f_{n2}\psi_{n2})]-\frac{1}{2}[(\psi_{n1}f_{n2}-\psi_{n2}f_{n1})(h_{n1}g_{n2}+h_{n2}g_{n1})+(\psi_{n1}f_{n2}+\psi_{n2}f_{n1})(h_{n1}g_{n2}-h_{n2}g_{n1})]=$$

$$=-C_n\psi_{n1}\psi_{n2}\left[\left(\frac{\beta_n}{\dot{a}(\xi_n)}\varphi_{n1}+\alpha_n g_{n1}\right)g_{n2}-\left(\frac{\beta_n}{\dot{a}(\xi_n)}\varphi_{n2}+\alpha_n g_{n2}\right)g_{n1}\right]=$$

$$=-C_n\psi_{n1}\psi_{n2}\frac{\beta_n}{\dot{a}(\xi_n)}\omega_n(t). \qquad (12)$$

(12) tenglikni x bo'yicha integrallasak

$$\int_{-\infty}^{\infty}(f_{n1}g_{n1}-f_{n2}g_{n2})(h_{n1}\psi_{n1}+h_{n2}\psi_{n2})dx=$$

$$-\frac{\beta_n}{\dot{a}(\xi_n)}\omega_n(t)\int_{-\infty}^{\infty}C_n\psi_{n1}\psi_{n2}dx=$$

$$=-\frac{\beta_n}{\dot{a}(\xi_n)}\frac{\omega_n(t)}{C_n}\int_{-\infty}^{\infty}\phi_{n1}\phi_{n2}dx=-\frac{i}{2}\beta_n(t)\omega_n(t) \qquad (13)$$

Shunday qilib, biz quyidagi

$\dfrac{dC_n}{dt}=[8i\xi_n^3+i\beta_n(t)\omega_n(t)]C_n,\quad n=\overline{1,N}$ tenglikka ega bo'lamiz.

Endi t ga nisbatan evolyutsiyalarni hisoblaylik.

Agar $\xi_k\neq\xi_n$ bo'lsa

$$\int_{-\infty}^{\infty}(f_{k1}g_{k1}-f_{k2}g_{k2})(\varphi_{n1}^2+\varphi_{n2}^2)dx=$$

$$=\frac{1}{-2i(\xi_n+\xi_k)}\int_{-\infty}^{\infty}\frac{d}{dx}[(f_{k1}\varphi_{n1}+\varphi_{n2}f_{k2})(\varphi_{n1}g_{k1}+\varphi_{n2}g_{n2})]dx+$$

$$+\frac{1}{2i(\xi_n-\xi_k)}\int_{-\infty}^{\infty}\frac{d}{dx}[(f_{k2}\varphi_{n1}-\varphi_{n2}f_{k1})(\varphi_{n1}g_{k2}-\varphi_{n2}g_{n1})]dx=$$

$$=\frac{-1}{2i(\xi_n+\xi_k)}\lim_{R\to\infty}[(f_{k1}\varphi_{n1}+\varphi_{n2}f_{k2})(\varphi_{n1}g_{k1}+\varphi_{n2}g_{n2})|_{-R}^{R}]+$$

$$+\frac{1}{2i(\xi_n-\xi_k)}\lim_{R\to\infty}[(f_{k2}\varphi_{n1}-\varphi_{n2}f_{k1})(\varphi_{n1}g_{k2}-\varphi_{n2}g_{k1})|_{-R}^{R}]=0$$

ga ega bo'lamiz.

Agar $\xi_k=\xi_n$ bo'lsa

$$\int_{-\infty}^{\infty}(f_{n1}g_{n1}-f_{n2}g_{n2})(\varphi_{n1}^2+\varphi_{n2}^2)dx=$$

$$=\frac{1}{-4i\xi_n}\int_{-\infty}^{\infty}\frac{d}{dx}[(f_{n1}\varphi_{n1}+\varphi_{n2}f_{n2})(\varphi_{n1}g_{n1}+\varphi_{n2}g_{n2})]dx-$$

$$-\frac{1}{2}\int_{-\infty}^{\infty}[(f_{n1}\varphi_{n2}-\varphi_{n1}f_{n2})(\varphi_{n1}g_{n2}+\varphi_{n2}g_{n1})+$$

$$+(f_{n2}\varphi_{n1}+\varphi_{n2}f_{n1})(\varphi_{n1}g_{n2}-\varphi_{n2}g_{n1})]dx=$$

$$=-\frac{1}{2}\int_{-\infty}^{\infty}2\varphi_{n1}\varphi_{n2}(f_{n1}g_{n2}-f_{n2}g_{n1})dx=-\omega_n(t)\int_{-\infty}^{\infty}\varphi_{n1}\varphi_{n2}dx.$$

$$\frac{d\xi_n}{dt}=i\omega_n(t),\quad n=\overline{1,N}$$

Shunday qilib, biz quyidagi teoremani isbotladik.

Teorema 1: Agar $u(x,t)$, $f_k(x,t)$, $g_k(x,t)$, $k=\overline{1,N}$, funksiyalari (1) – (5) funksiyalar sinfidagi muammoning yechimi bo'lsa, u holda $L(t)$ operatorning $u(x,t)$ potentsial o'zgarishiga ega sochilish ma'lumotlari quyidagicha bo'ladi.

$$\frac{d\xi_n}{dt}=i\omega_n(t),\qquad n=\overline{1,N},$$

$$\frac{dC_n}{dt}=[8i\xi_n^3+i\beta_n(t)\omega_n(t)]C_n,\quad n=\overline{1,N},$$

$$\frac{dr^+}{dt}=\left[8i\xi^3+\sum_{k=1}^{N}i\omega_k(t)\left(\frac{1}{\xi+\xi_k}+\frac{1}{\xi-\xi_k}\right)\right]r^+,\quad(\operatorname{Im}\xi=0),$$

(7)tengliklardan aniqlanadi .

Xulosa

Ushbu maqolada nochiziqli mKdF tenglamasini oddiy xos qiymatlarga mos keladigan moslangan manba bilan integratsiyalashuvi "tez kamayuvchi" funksiyalar sinfida o'rganilgan. Ya'ni, oddiy xos qiymatlarga ega bo'lgan o'z - o'ziga qo'shma bo'lmagan Dirak operatorining sochilish ma'lumotlarining evolyutsiyasi olingan bo'lib, uning potensiali moslangan manba bilan mKdF tenglamasining yechimi hisoblanadi.

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