VANDERMOND DETERMINANTINI HISOBLASH

Turdiboyev Sanjar Sobirjon o'g'li
JDPU, Matematika va informatika fakulteti, katta o'qituvchisi

Annotatsiya. *Ma`lumki, determinantlar qiymatini hisoblashning turli xil yo`llari(usullari) mavjud. Ushbu usullarda determinantning xossalaridan foydalanish, minorlar va algebraik to`ldiruvchilar yordamida ustun yoki satr elementlari bo`yicha yoyish ko`zda tutiladi. Umumiy jihatdan olib qaralganda, determinanatlar turli xil bo`lgani uchun, ularni hisoblash usullari ham shunga yarasha ko`p va turlichadir. Ba`zan faqatgina, maxsus determinantlarning qiymatini hisoblash usullari oldindan berilishi mumkin. Hozir esa, quyida bir maxsus determinantning qiymatini hisoblash usulini ko`rib chiqamiz.*

Kalit so'zlar: *Determinant, minorlar, algebraik to`ldiruvchilar, ustun lementlar, satr elementlar*

a_1 , a_2, ... , a_n elementlardan iborat quyidagi n-darajali determinantni, ya`ni **Vandermond** determinantini hisoblaymiz:

$$V_n=\begin{vmatrix} 1 & a_1 & a_1^2 & a_1^3 \ldots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & a_2^3 \ldots & a_2^{n-1} \\ \ldots & \ldots & \ldots & \ldots\ldots & \ldots \\ 1 & a_n & a_n^2 & a_n^3 \ldots & a_n^{n-1} \end{vmatrix} \qquad (1)$$

Birinchi ustunni $-a_1$ ga ko`paytiramiz va ikkinchi ustunga qo`shamiz[4]. Bu jarayonda, ya`ni ustunlarni $-a_1$ ga ko`paytirib o`zidan keying ustunga qo`shayotganimizda, (1) berilgan determinantning ustunlarini qaraymiz.

$$V_n=\begin{vmatrix} 1 & 0 & a_1^2 & a_1^3 \ldots & a_1^{n-1} \\ 1 & a_2-a_1 & a_2^2 & a_2^3 \ldots & a_2^{n-1} \\ 1 & a_3-a_1 & a_3^2 & a_3^3 \ldots & a_3^{n-1} \\ \ldots & \ldots & \ldots & \ldots\ldots & \ldots \\ 1 & a_n-a_1 & a_n^2 & a_n^3 \ldots & a_n^{n-1} \end{vmatrix}$$

Keyingi qadamda, ikkinchi ustunni ham $-a_1$ ga ko`paytiramiz va uchinchi ustunga qo`shamiz:

$$V_n=\begin{vmatrix} 1 & 0 & 0 & a_1^3 & \dots & a_1^{n-1} \\ 1 & a_2-a_1 & a_2(a_2-a_1) & a_2^3 & \dots & a_2^{n-1} \\ 1 & a_3-a_1 & a_3(a_3-a_1) & a_3^3 & \dots & a_3^{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & a_n-a_1 & a_n(a_n-a_1) & a_n^3 & \dots & a_n^{n-1} \end{vmatrix}$$

Huddi shu kabi, qolgan barcha ustunlarni ham $-a_1$ ga ko`paytiramiz va o`zidan keying ustunga qo`shamiz:

$$V_n=\begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & a_2-a_1 & a_2(a_2-a_1) & a_2^2(a_2-a_1)\dots & & a_2^{n-2}(a_2-a_1) \\ 1 & a_3-a_1 & a_3(a_3-a_1) & a_3^2(a_3-a_1)\dots & & a_3^{n-2}(a_3-a_1) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & a_n-a_1 & a_n(a_n-a_1) & a_n^2(a_n-a_1)\dots & & a_n^{n-2}(a_n-a_1) \end{vmatrix}$$

Endi esa ushbu determinantni 1-satr elementlari bo`yicha yoyamiz[3], va quyidagi (n-1)- tartibli determinantga ega bo`lamiz:

$$V_n=\begin{vmatrix} a_2-a_1 & a_2(a_2-a_1) & a_2^2(a_2-a_1)\dots & a_2^{n-2}(a_2-a_1) \\ a_3-a_1 & a_3(a_3-a_1) & a_3^2(a_3-a_1)\dots & a_3^{n-2}(a_3-a_1) \\ \dots & \dots & \dots\quad\dots & \dots \\ a_n-a_1 & a_n(a_n-a_1) & a_n^2(a_n-a_1)\dots & a_n^{n-2}(a_n-a_1) \end{vmatrix};$$

Determinantning xossasiga ko`ra, bir satr elementlarini biror songa ko`paytirilsa, determinant qiymati shuncha martaga ortadi[5]. Demak,

$$V_n=\begin{vmatrix} a_2-a_1 & a_2(a_2-a_1) & a_2^2(a_2-a_1)\dots & a_2^{n-2}(a_2-a_1) \\ a_3-a_1 & a_3(a_3-a_1) & a_3^2(a_3-a_1)\dots & a_3^{n-2}(a_3-a_1) \\ \dots & \dots & \dots\quad\dots & \dots \\ a_n-a_1 & a_n(a_n-a_1) & a_n^2(a_n-a_1)\dots & a_n^{n-2}(a_n-a_1) \end{vmatrix}=$$

$$=(a_2-a_1)\begin{vmatrix} 1 & a_2 & a_2^2 & \dots & a_2^{n-2} \\ a_3-a_1 & a_3(a_3-a_1) & a_3^2(a_3-a_1)\dots & & a_3^{n-2}(a_3-a_1) \\ \dots & \dots & \dots & \dots & \dots \\ a_n-a_1 & a_n(a_n-a_1) & a_n^2(a_n-a_1)\dots & & a_n^{n-2}(a_n-a_1) \end{vmatrix};$$

ga ega bo`lamiz va determinantni shu tarzda soddalashtirishda davom etsak,

$$V_n=(a_2-a_1)\begin{vmatrix} 1 & a_2 & a_2^2 & \dots & a_2^{n-2} \\ a_3-a_1 & a_3(a_3-a_1) & a_3^2(a_3-a_1)\dots & & a_3^{n-2}(a_3-a_1) \\ \dots & \dots & \dots & \dots & \dots \\ a_n-a_1 & a_n(a_n-a_1) & a_n^2(a_n-a_1)\dots & & a_n^{n-2}(a_n-a_1) \end{vmatrix}=$$

$$=(a_2-a_1)(a_3-a_1)(a_4-a_1)\dots(a_n-a_1)\begin{vmatrix} 1 & a_2 & a_2^2 & \dots & a_2^{n-2} \\ 1 & a_3 & a_3^2 & \dots & a_3^{n-2} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_n & a_n^2 & \dots & a_n^{n-2} \end{vmatrix}$$

hosil bo`ladi. Ushbu hosil qilingan ifodaning o`ng tarafida, yuqorida ta`kidlaganimizdek, (n-1)-tartibli $V_{n-1}$ determinant hosil bo`ldi. Bu determinant elementlarini a_2 , a_3, … , a_n elementlardan iborat ekanligini inobatga olgan holda, V_n determinantga nisbatan qo`llagan amallarimizni, huddi shu ketma-ketlikda $V_{n-1}$ determinantga ham qo`llasak,

$$V_{n-1}=(a_3-a_2)(a_4-a_2)(a_5-a_2)\dots(a_n-a_2)\begin{vmatrix} 1 & a_3 & a_3^2 & \dots & a_3^{n-3} \\ 1 & a_4 & a_4^2 & \dots & a_4^{n-3} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_n & a_n^2 & \dots & a_n^{n-3} \end{vmatrix}$$

ekani kelib chiqadi. Tenglikning o`ng tomonida, (n-2)- tartibli va $a_3$ , $a_4$, … , $a_n$ elementlardan iborat bo`lgan V_{n-2} determinantning hosil bo`lganini ko`rishimiz mumkin. Bu determinant uchun ham yuqoridagi V_n va V_{n-1} determinantlarga qilgan ishlarimizni takrorlasak, shu kabi hosil bo`lgan determinantlar uchun ham shunday davom etsak,

$$V_n=(a_2-a_1)(a_3-a_1)(a_4-a_1)\dots(a_n-a_1)(a_3-a_2)(a_4-a_2)(a_5-a_2)\dots$$
$$\dots(a_n-a_2)\dots(a_n-a_{n-1})=\prod_{i>j>1}^{n}(a_i-a_j)$$

ga ega bo`lamiz. Bundan kelib chiqadiki,

$$V_n=\begin{vmatrix} 1 & a_1 & a_1^2 & a_1^3 & \dots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & a_2^3 & \dots & a_2^{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & a_n & a_n^2 & a_n^3 & \dots & a_n^{n-1} \end{vmatrix}=\prod_{i>j>1}^{n}(a_i-a_j)$$

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