CHIZIQLI OPERATOR

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Agar R^3da chiziqli $\tilde{A}$ operator $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda o'zining $A = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix}$ matritsasi bilan berilgan bo'lsa, $\vec{x} = 4\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning $y=A(x)$ aksini toping.

Y=AX formulaga binoan,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix} * \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -13 \\ -18 \end{bmatrix}$$

Demak, $y = 10e_1 - 13e_2 - 18e_3$

$\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ operator $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ matritsaga ega.

$e_1 = e_1 - 2e_2,\quad e_2 = 2e_2 + e_2$ bazisida $\tilde{A}$ operatorining matritsasini toping. O'tish matritsasi

$$C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

ning teskari matrisasi

$$C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

Demak,

$$B = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 8 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}$$

Chizqli $\tilde{A}$ operator $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

Xarakteristik tenglama tuzamiz:

$$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix} = 0, \quad \lambda^2 - 2\lambda - 35 = 0; \quad \lambda_1 = -5, \quad \lambda_2 = 7$$

$\lambda_1=-5$ ga tegishli $X^{(1)}=(X_1,X_2)$ xos vektorni topamiz. Buning uchun quyidagi tenglamani echamiz:

$$\lambda_1 = -5 \quad (A - \lambda E)\cdot x = 0 \qquad \begin{pmatrix} 6 & 4 \\ 9 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_2 = -1{,}5x_1$$

agar $x_1=C$ deb olsak $x_2=-1.5C$, $X^{(1)}=(C;-1.5C)$ vektorlar har qanday $C\neq 0$ uchun A operatorni xos qiymati $\lambda_1=-5$ ga tegishli xos vector bo'ladi. Xuddi shunday $\lambda_2=7$ xos qiymati uchun A operatorni xos vektorlarni

$$X^{(2)} = \left(\frac{2}{3}C_1, C_1\right), \qquad C_1 \neq 0$$

vektorlar tashkil etadi.

Chiziqli operatorning $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsasini diogonal ko'rinishiga keltiring.

$A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matrisa bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarning 3-misolda topilgan: $\lambda_1=-5$ $\lambda_2=7$

$$X^{(1)} = (C;-1{,}5C); \quad X^{(2)} = \left(\frac{2}{3}C_1, C_1\right); \quad X^{(1)} \ va \ X^{(2)}$$

vektorning koordinatalari proportsional emas, shuning uchun $X^{(1)}$ va $X^{(2)}$ vektorlar chiziqli erkli. Demak, $X^{(1)}$ va $X^{(2)}$ bazisda A-matritsaning diagonal ko'rinishi:

$$A^* = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad ёки \quad A^* = \begin{pmatrix} -5 & 0 \\ 0 & 7 \end{pmatrix}$$

Buni tekshirish uchun bazis vektorlar sifatida $X^{(1)}=(2;\ -3)$, $X^{(2)}=(4;\ 6)$ vektorlarni olsak, yangi bazisga o'tkazuvchi o'tish matritsa C ning ko'rinishi: $\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix}$ bo'ladi.

Dioganal matrtisa:

$$A^* = C^{-1}AC = \begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix} = \frac{1}{24}\begin{pmatrix} 6 & -4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix} = \frac{1}{24}\begin{pmatrix} -30 & 20 \\ 21 & 14 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix} = \frac{1}{24}\begin{pmatrix} -120 & 0 \\ 0 & 168 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & 7 \end{pmatrix}$$

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