AJOYIB IRRATSIONAL FUNKTSIYALARNI INTEGRALLASH

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Annotatsiya: *Ushbu maqola o'qituvchi va o'quvchilarga metodik tavsiya sifatida yozilgan. Matematikaning asosiy bo'limlaridan biri integrallash haqida ma'lumot beriladi. O'quvchi bu* $\int x^m(a+bx^n)^p$ $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ $\int \frac{dx}{(x+\alpha)\sqrt{ax^2+bx+c}}$ *misollarda o'rganish natijasida integrallash mavzusiga qiziqishi ortadi. Biz ushbu maqolada qiyin uchta misolni yechimlarini keltrib otganmiz va shu mavzu yuzasidan misollar ham ko'rsatishga harakat qildik. integrallash oid dars jarayonida o'qituvchi maqoladan ko'rgazma sifatida foydalansa bo'ladi. Maqola matematikani o'qitish samaradorligini oshirishda xizmat qiladi.Bu maqolamiz sizlarga manzur bo'ladi degan umiddamiz.*

Kalit so'zlar: *Irratsional funktsiya, kvadrat uch had, to`la kvadirat, o`zgaruvchini almashtirish, N`yuton Binomi, ratsional funksiya*

Ko`p hollarda o`zgaruvchini almashtirish bilan ratsional funktsiyalarni integrallashga keltiriladi. Bunday irratsional funktsiyalarning ayrimlarini qaraymiz.

1. $\int x^m(a+bx^n)^p$ ko`rinishdagi integralni hisoblash talab etilsin, bunda $m,n,p$ ratsional sonlar, $a$ va b lar no`ldan farqli o`zgarmaslar.

1) p butun son bo`lsa, N`yuton Binomi bo`yicha yoyish bilan integrallanadi;

$a+bx^n=t^s$ ratsionallashtiriladi, bunda s p kasrning maxraji;

3) $\frac{m+1}{n}+p$ butun bo`lsa, $ax^{-n}+b=t^s$ almashtirish olinib, ratsional funktsiyaga keltiriladi.

1-misol. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx$ integralni toping.

Yechish. Integralni $\int x^{-2/3}(1+x^{1/3})dx$ ko`rinishida yozib,

$m=-\frac{2}{3}$, $n=\frac{1}{3}$, $p=\frac{1}{2}$, $\frac{m+1}{n}=\frac{-2/3+1}{1/3}=\frac{1/3}{1/3}=1$ bo`lganligi uchun $(1+x^{1/3})=t^2$ almashtirish olsak, $x^{1/3}=t^2-1$, $\frac{1}{3}x^{-2/3}dx=2tdt$, $x^{-2/3}dx=6tdt$ bo`ladi. Bularni berilgan integralga qo`ysak, $\int x^{-2/3}(1+x^{1/3})^{1/2}dx=\int t\cdot 6tdt=6\int t^2dt=6\cdot\frac{t^3}{3}+C=2(1+\sqrt[3]{x})^{3/2}+C$ bo`ladi.

2-misol. $\int\frac{dx}{\sqrt[4]{1+x^4}}$ integralni hisoblang.

Yechish. $\int\frac{dx}{\sqrt[4]{1+x^4}}=\int x^0(1+x^4)^{-1/4}dx$, $m=0$, $n=4$, $p=-\frac{1}{4}$ $\frac{m+1}{n}+p=\frac{1}{4}-\frac{1}{4}=0$ (butun) bo`lganligi uchun $x^{-4}+1=t^4$ almashtirish olsak,

$x=(t^4-1)^{-1/4}$, $dx=-t^3(t^4-1)^{-5/4}dt$, $\sqrt[4]{1+x^4}=t(t^4-1)^{-1/4}$ bo`ladi.

Demak, $\int\frac{dx}{\sqrt[4]{1+x^4}}=-\int\frac{t^2dt}{t^4-1}=\frac{1}{4}\int\left(\frac{1}{1+t}-\frac{1}{t-1}\right)dt-\frac{1}{2}\int\frac{dt}{t^2+1}=\frac{1}{4}\ln\frac{|t+1|}{t-1}-\frac{1}{2}arctgt+C$, $t=\sqrt[4]{1+x^4}$ bo`lganligi uchun,

$$\int\frac{dx}{\sqrt[4]{1+x^4}}=\frac{1}{4}\ln\frac{\left|\sqrt[4]{1+x^4}\right|+1}{\sqrt[4]{1+x^4}-1}-\frac{1}{2}arctg\sqrt[2]{1+x^4}+1+C$$ bo`ladi.

2. $\int\frac{dx}{\sqrt{ax^2+bx+c}}$ ko`rinishdagi integralni qaraymiz.

Bunday ko`rinishdagi ifodalarni integrallash kvadrat uch haddan to`la kvadrat ajratish bilan $\int\frac{du}{\sqrt{a^2-u^2}}$ yoki $\int\frac{du}{\sqrt{a^2+u^2}}$ jadval integrallaridan biriga keltiriladi.

3-misol. $\int\frac{dx}{\sqrt{x^2+2x+5}}$ integralni hisoblang.

Yechish. $x^2+2x+5=x^2+2x+1+4=(x+1)^2+4$ to`la kvadrat ajratib, $x+1=u$ desak, $\int\frac{dx}{\sqrt{x^2+2x+5}}=\int\frac{du}{\sqrt{u^2+4}}=\ln\left|u+\sqrt{u^2+4}\right|+C=\ln\left|(x+1)+\sqrt{x^2+2x+5}\right|+C$ bo`ladi.

3) $\int \frac{dx}{(x+\alpha)\sqrt{ax^2+bx+c}}$ ko`rinishdagi integral, $\frac{1}{x+\alpha}=t$ almashtirish orqali 2. ko`rinishdagi integralga keltiriladi.

4-misol. $\int \frac{dx}{(x+1)\sqrt{x^2+2x+2}}$ integralni hisoblang.

Yechish. $\frac{1}{x+1}=t$ bilan almashtirsak,

$$t(x+1)=1,\ tx+t=1,\ tx=1,\ x=\frac{1-t}{t},\ dx=-\frac{1}{t^2}dt$$

bo`lib,

$$\int \frac{dx}{(x+1)\sqrt{x^2+2x+2}}=\int \frac{t\left(\frac{-1}{t^2}\right)dt}{\sqrt{\left(\frac{1-t}{t}\right)^2+2\frac{1-t}{t}+2}}=-\int \frac{dt}{t\sqrt{\frac{1-2t+t^2}{t^2}+\frac{2-2t}{t}+2}}=$$

$$=-\int \frac{dt}{\sqrt{1-2t+t^2+2t}}=-\int \frac{dt}{\sqrt{t^2+1}}$$

bu jadval integraldir. (Oxirgi integralni mustaqil bajarishni o`quvchiga havola qilamiz).

1. Ushbu irratsional funktsiyalarni integrallang.

1) $\int \frac{2x+1}{\sqrt{x^2-4x+1}}dx$; 2) $\int \frac{2x-3}{\sqrt{8-2x-x^2}}dx$; 3) $\int \frac{7x-5}{\sqrt{\sqrt{5+2x-x}^2}}dx$; 4) $\int \frac{dx}{\sqrt[3]{(2x+1)^2}-\sqrt{2x+1}}$; 5) $\int \frac{\sqrt{x}+3}{x\left(\sqrt{x}+\sqrt[3]{x}\right)}dx$.