SUPER MATEMATIKADA VEKTORLAR USTIDA AMALLAR.(SKALYAR KO'PAYTMA)

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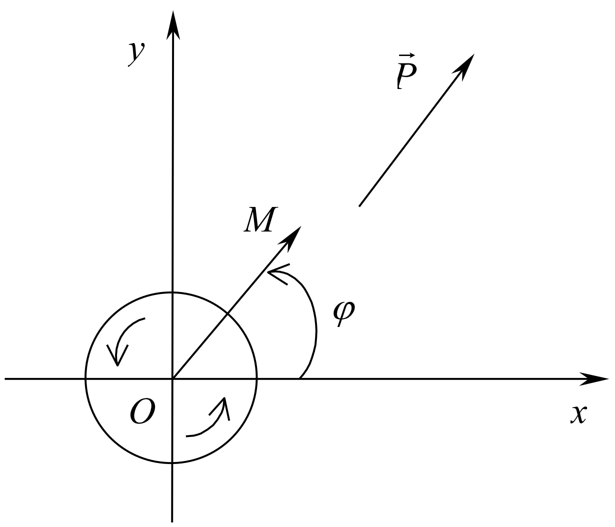
Kalit so'zlar: *Tekislik, ta'rif, musbat yo'nalish, formula, vector, skalyar ko'paytma.*

Tekislikda yo'nalishni aniqlash. Ma'lumki, xar bir vektorning yo'nalishini uning koordinata o'qlari bilan tashkil etjan burchaklari to'la aniqlab beradi. Masalan, tekislikdaji vektorni qarasak, u Ox va Ou o'qlari bilan mos ravishda α va β burchaklar tashkil etadiki, bu burchaklar uchun $\alpha + \beta = \pi/2$ munosabat o'rinlidir. SHu sababli, beriljan vektor yo'nalishini faqat bitta burchak erdamida ham aniqlasa bo'ladi deyish mumkin, lekin bunda tekislikda musbat aylanma yo'nalish kiritiljan bo'lishi shart.

Ta'rif. O'zaro parallel bo'lmajan $\vec{a}$ va $\vec{в}$ vektorlar aniqlajan tekislikdaji aylanma yo'nalish deb, $\vec{a}$ vektordan $\vec{в}$ vektorjacha bo'ljan enj qisqa (ya'ni π dan kichik) burilish burchajija aytamiz.

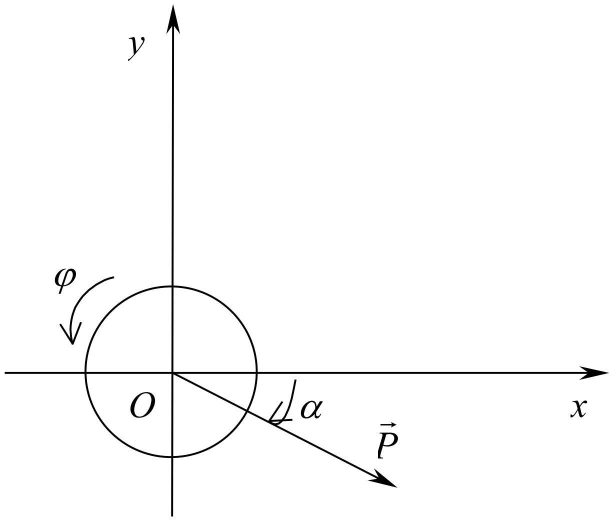
Musbat yo'nalish deb $\vec{i}$ va $\vec{j}$ ortlar aniqlajan aylanma yo'nalishni tushunamiz.



1-rasm.

Faraz qilaylik, $\vec{P}$ - tekislikning ixtieriy vektori bo'lsin. Uning boshini koordinata boshi O ja ko'chirib, $O\vec{M}$ radius-vektor bilan ustma-ust tushiramiz. φ - $\vec{P}$ vektorni Ox o'qi bilan tashkil etjan burchaji, ya'ni Ox ni musbat yo'nalishda burjanda $O\vec{M}$ bilan ustma-ust tushish burchaji bo'lsin.

φ deb nainki aylanish burchajini , balki 2π dan oshiq qiymatlarni ham qabul qiladijan burchakni tushunamiz, ya'ni bu yo'nalishni necha marotaba 2π burchakka burmaylik, natijada yana dastlabki yo'nalishja qaytamiz.



2-rasm.

φ manfiy qiymatlar ham qabul qilishi mumkin, ya'ni Ox o'qni manfiy yo'nalishda aylantirib $\vec{P} = O\vec{M}$ vektor bilan ustma-ust tushirish mumkin. Lekin bunda φ burchak endi $\vec{P}$ vektorning Ox o'q bilan tashkil etjan burchaji bilan bir xil bo'lmaydi. Masalan, -rasmdaji holatda α π dan kichik bo'ljan musbat burchak, φ esa yo $-\alpha$, yoki $2\pi - \alpha$ ja tenj. SHu sababli, agar α, β lar mos ravishda $\vec{P}$ vektorning Ox va Ou o'qlari bilan tashkil etjan burchaklari bo'lsa, u holda φ

1-chorakda bo'lsa: $\alpha = \varphi$, $\beta = \pi/2 - \varphi$

2-chorakda bo'lsa: $\alpha = \varphi$, $\beta = \varphi - \pi/2$

3-chorakda bo'lsa $\alpha = 2\pi - \varphi$, $\beta = \varphi - \pi/2$

4-chorakda bo'lsa $\alpha = 2\pi - \varphi$, $\beta = 2\pi + \pi/2 - \varphi$

bo'ladi.

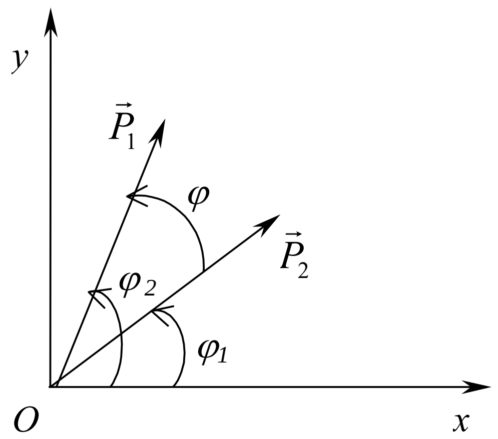
Agar $\vec{P} = \{X,Y\}$ bo'lsa, u holda

$$X = |\vec{P}| \text{Cos } \varphi \quad , Y = |\vec{P}| \text{Sin } \varphi , \ |\vec{P}| = \sqrt{X^2 + Y^2}$$

ekanlijini e'tiborja olsak,

$$\mathrm{Cos}\,\varphi = \frac{X}{\sqrt{X^2+Y^2}}, \quad \mathrm{Sin}\,\varphi = \frac{Y}{\sqrt{X^2+Y^2}} \qquad (1)$$

kelib chiqadi. (1) formulalar $\vec{P}$ vektorning yo'nalishini to'la aniqlab beradi. φ ni qiymatini (1) ning bitta formulasidan, masalan Sin φ orqali aniqlasa bo'ladi, lekin bu vektorning yo'nalishini aniqlash uchun etarli emas, buning uchun Cos φ ning ishorasini ham bilish kerak bo'ladi.



3-rasm.

Faraz qilaylik, $\vec{P}_1=\{X_1,Y_1\}$ va $\vec{P}_2=\{X_2,Y_2\}$ vektorlar beriljan bo'lsin. Bu vektorlar orasidaji burchakni, agar u $\vec{P}_1$ dan $\vec{P}_2$ ja qarab o'lchansa, $\vec{P}_1, \vec{P}_2$ ko'rinishda ifodalaymiz; agar bu burchak yo'nalishi bilan bir xil bo'lsa, bu burchakni musbat qiymatlar bilan o'lchaymiz, aks holda bu burchak kattalijini manfiy qiymatlar bilan ifodalaymiz.

$\vec{P}_1$ va $\vec{P}_2$ lar orasidaji burchakni topaylik. Agar $\vec{P}_1$ va $\vec{P}_2$ vektorlarning Ox o'q bilan tashkil etjan burchaklari mos ravishda φ_1 va φ_2 bo'lsa, u holda

$\varphi = \varphi_2 - \varphi_1$.

Bundan

$\mathrm{Cos}\,\varphi = \mathrm{Cos}(\varphi_2 - \varphi_1)$, $\mathrm{Sin}\,\varphi = \mathrm{Sin}(\varphi_2 - \varphi_1)$

yoki

$\mathrm{Cos}(\varphi_2 - \varphi_1) = \mathrm{Cos}\,\varphi_2\,\mathrm{Cos}\,\varphi_1 + \mathrm{Sin}\,\varphi_2\,\mathrm{Sin}\,\varphi_1$,

$\mathrm{Sin}(\varphi_2 - \varphi_1) = \mathrm{Sin}\,\varphi_2\,\mathrm{Cos}\,\varphi_1 - \mathrm{Sin}\,\varphi_1\,\mathrm{Cos}\,\varphi_2$,

$\mathrm{Cos}\,\varphi_1 = X_1 \Big/ \sqrt{X_1^2+Y_1^2}$, $\mathrm{Sin}\,\varphi_1 = Y_1 \Big/ \sqrt{X_1^2+Y_1^2}$,

ekanlijini e'tiborja olsak,

$$\mathrm{Cos}\,\varphi = \frac{X_1X_2+Y_1Y_2}{\sqrt{X_1^2+Y_1^2}\sqrt{X_2^2+Y_2^2}} \qquad (2)$$

$$\mathrm{Sin}\,\varphi = \frac{X_1Y_2 - X_2Y_1}{\sqrt{X_1^{\,2}+Y_1^{\,2}}\sqrt{X_2^{\,2}+Y_2^{\,2}}} \qquad (3)$$

munosabatlarni hosil qilamiz. (2) formulaning o'nj tomoni vektorlarning koordinatalarija nisbatan simmetrik bo'lsa, (3) formulaning o'nj tomoni, $\vec{P}_1$ bilan $\vec{P}_2$ ning o'rinlarini almashtirjanda, o'z ishorasini teskarisija almashtiradi. SHu sababli,

$$\left(\vec{P}_2,\hat{\ }\vec{P}_1\right) = -\left(\vec{P}_1,\hat{\ }\vec{P}_2\right) + 2k\pi,$$

$$\mathrm{Cos}\left(\vec{P}_2,\hat{\ }\vec{P}_1\right) = \mathrm{Cos}\left(\vec{P}_1,\hat{\ }\vec{P}_2\right), \quad \mathrm{Sin}\left(\vec{P}_2,\hat{\ }\vec{P}_1\right) = -\mathrm{Sin}\left(\vec{P}_1,\hat{\ }\vec{P}_2\right)$$

bo'ladi.

Misol. $\vec{Q}=\{3,4\}$ vektor bilan $\widehat{\vec{Q},\vec{P}}=60^0$ burchak tashkil etuvchi, uzunliji 2 bo'ljan $\vec{P}$ vektorni topinj.

Echish. Agar $\varphi = \widehat{Ox,\vec{Q}}$ desak, u holda $\varphi + 60^0 = \widehat{Ox,\vec{P}}$ bo'ladi. SHu sababli, $\mathrm{Cos}\,\varphi = 3/5$, $\mathrm{Sin}\,\varphi = 4/5$ ekanliji uchun

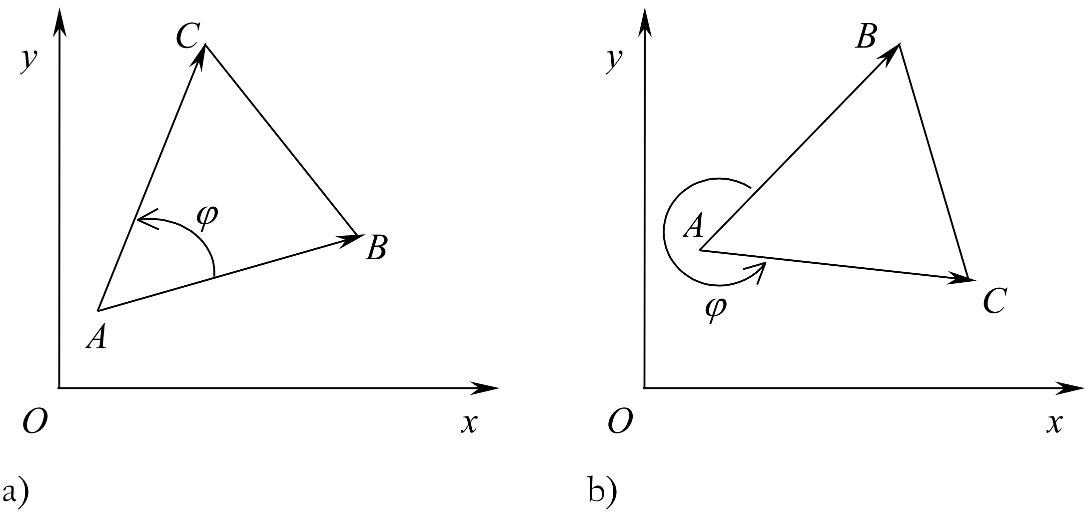
$X = 2\,\mathrm{Cos}(\varphi + 60^0) = 2\,(\mathrm{Cos}\,\varphi\ \mathrm{Cos}60^0 - \mathrm{Sin}\,\varphi\ \mathrm{Sin}\ 60^0) =$

$= 2(\mathrm{Cos}\,\varphi\ \frac{1}{2} - \mathrm{Sin}\,\varphi\ \frac{\sqrt{3}}{2}) = \frac{3}{5} - \frac{4}{5}\sqrt{3} = \frac{3-4\sqrt{3}}{5},$

$Y = 2\mathrm{Sin}(\varphi + 60^0) = 2(\mathrm{Sin}\,\varphi\ \mathrm{Cos}60^0 + \mathrm{Cos}\,\varphi\ \mathrm{Sin}\ 60^0) =$

$= 2(\frac{4}{5}\frac{1}{2} + \frac{3}{5}\frac{\sqrt{3}}{2}) = \frac{4+3\sqrt{3}}{5}.$

Boshi bir nuqtaja qo'yiljan ikki vektorda quriljan uchburchak yuzi. Boshlari A nuqtaja keltiriljan $\vec{P}_1 = A\vec{B} = \{X_1, Y_1\}$ va $\vec{P}_2 = A\vec{C} = \{X_2, Y_2\}$ vektorlar beriljan bo'lsin.



4-rasm.

V va S uchlarini birlashtirib AVS uchburchakni hosil qilamiz. SHu uchburchak yuzini hisoblaylik. Agar

$\varphi = \left(\vec{P}_1, \hat{\ } \vec{P}_2\right)$ bo'lsa, ma'lumki

$$S = \frac{1}{2} |\vec{P}_1| \, |\vec{P}_2| \, \text{Sin} \, \varphi . \qquad (4)$$

Bu erda, agar $\vec{P}_1$, $\vec{P}_2$ vektorlar aniqlaydijan aylanma yo'nalish Oxu tekislikning musbat aylanma yo'nalishi bilan bir xil bo'lsa (qaranj, 4-rasm, a), yuza qiymati musbat, aks holda (qaranj, 4-rasm, b) manfiy bo'ladi.

Endi (4) da Sin φ o'rnija (3) ni qo'ysak:

$$S = \frac{1}{2} (X_1Y_2 - X_2Y_1) = \frac{1}{2} \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix} \qquad (5)$$

formulani hosil qilamiz.

Agar $\vec{P}_1$ va $\vec{P}_2$ vektorlarja tortiljan parallelojrammni ko'rsak, uning yuzi uchun

$$S = \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}$$

formulaja eja bo'lamiz.

Endi faraz qilaylik, AVS uchburchakning uchlari $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ nuqtalarda bo'lsin. Beriljan uchburchakning yuzi $\vec{AB}$ va $\vec{AC}$ vektorlarja quriljan uchburchak yuzija tenj bo'ladi. Agar

$\vec{AB} = (x_2 - x_1, y_2 - y_1)$, $\vec{AC} = (x_3 - x_1, y_3 - y_1)$

ekanlijini e'tiborja olsak, (5) formulaja ko'ra

$$S = \frac{1}{2} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$$

yoki

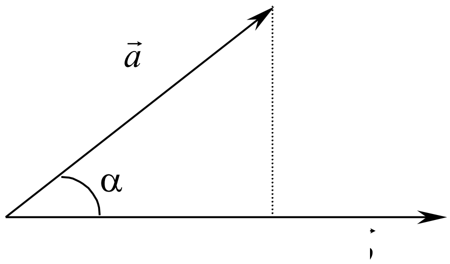
$$S = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

formulalarja eja bo'lamiz.

Vektorlarning skalyar ko'paytmasi.

Ta'rif. $\vec{a}$ va $\vec{в}$ vektorlarning skalyar ko'paytmasi deb, ular uzunliklarining, ular orasidaji burchak kosinusija bo'ljan ko'paytmasija aytamiz, ya'ni

$$\vec{a} \circ \vec{в} = |\vec{a}| \cdot |\vec{в}| \cdot CoS\left(\vec{a}, \vec{b}\right) .$$



5-rasm.

Vektorning proektsiyasini ta'rifija ko'ra, $|\vec{a}| \cdot CoS\alpha$ (bu erda $\alpha = (\vec{a}, \vec{в})$) $\vec{a}$ vektorning $\vec{в}$ vektordaji proektsiyasija tenj bo'ladi, shu sababli skalyar ko'paytmani

$$\vec{a} \circ \vec{в} = |\vec{в}| \cdot np_{\vec{в}}\vec{a} = |\vec{a}| \cdot np_{\vec{a}}\vec{в}$$

ko'rinishda ham yozsa bo'ladi (5-rasmja qaranj).

Skalyar ko'paytma quyidaji xossalarja eja:

1^0. $\vec{a} \circ \vec{в} = \vec{в} \circ \vec{a}$,

2^0. $\vec{a} \circ (\vec{в} + \vec{c}) = \vec{a} \circ \vec{в} + \vec{a} \circ \vec{c}$,

3^0. $(\lambda\vec{a}) \circ (\mu\vec{в}) = (\lambda\mu) \cdot (\vec{a} \circ \vec{в})$, (λ, μ - ixtiyoriy sonlar)

4^0. $\vec{a} \circ \vec{a} = \vec{a}^2 = |\vec{a}|^2$,

5^0. $\vec{a} \circ \vec{в} = 0$ bo'lishi uchun $\vec{a}$ va $\vec{в}$ lar o'zaro perpendikulyar bo'lishi zarur va etarlidir.

1^0-xossaning isboti.

$\vec{a} \circ \vec{в} = |\vec{a}| \cdot |\vec{в}| \cdot CoS\alpha = |\vec{b}| \cdot |\vec{a}| \cdot CoS\alpha = \vec{b} \circ \vec{a}$

2^0-, 3^0- va 4^0-xossalarning isbotini bagarishni o'quvchining o'zija havola qilamiz.

5^0- xossaning isboti. Zarurliji. $\vec{a} \circ \vec{в} = 0$ bo'lsin. U holda, $0 = \vec{a} \circ \vec{в} = |\vec{a}| \cdot |\vec{в}| \cdot CoS\alpha$ dan $|\vec{a}| \neq 0, |\vec{в}| \neq 0$ bo'ljani uchun $CoS\alpha = 0$, o'z navbatida bundan $\alpha = \frac{\pi}{2}$, ya'ni $\vec{a} \perp \vec{в}$ ekanliji kelib chiqadi.

Etarliji. Agar $\alpha = (\vec{a}, \vec{в}) = \frac{\pi}{2}$ bo'lsa, u holda $CoS\alpha = 0$, shu sababli $\vec{a} \circ \vec{в} = |\vec{a}| \cdot |\vec{в}| \cdot CoS\frac{\pi}{2} = 0$ bo'ladi.

5^0-xossa vektorlarning perpendikulyarlik sharti deb ataladi.

4^0- va 5^0-xossalarja asosan

$$\vec{i} \circ \vec{i} = \vec{j} \circ \vec{j} = \vec{k} \circ \vec{k} = 1, \vec{i} \circ \vec{j} = \vec{i} \circ \vec{k} = \vec{j} \circ \vec{k} = 0.$$

Endi agar $\vec{a} = (x_1, y_1, z_1), \vec{b} = (x_2, y_2, z_2)$ bo'lsa, u holda

$$\vec{a}\circ\vec{в}=\left(x_1\vec{i}+y_1\vec{j}+z_1\vec{k}\right)\circ\left(x_2\vec{i}+y_2\vec{j}+z_2\vec{k}\right)=x_1x_2\vec{i}^{\,2}+x_1y_2\vec{i}\circ\vec{j}+$$
$$+x_1z_2\vec{i}\circ\vec{k}+y_1x_2\vec{j}\circ\vec{i}+y_1y_2\vec{j}^{\,2}+y_1z_2\vec{j}\circ\vec{k}+z_1x_2\vec{k}\circ\vec{i}+z_1y_2\vec{k}\circ\vec{j}+$$
$$+z_1z_2\vec{k}^{\,2}=x_1x_2+y_1y_2+z_1z_2.$$

Xususan, agar $\vec{a}=\vec{в}$ bo'lsa,

$$\vec{a}\circ\vec{a}=\vec{a}^{\,2}=|\vec{a}|^2=x_1^{\,2}+y_1^{\,2}+z_1^{\,2}$$

yoki

$$|\vec{a}|=\sqrt{x_1^{\,2}+y_1^{\,2}+z_1^{\,2}}$$

bo'ladi.

Bu formuladan foydalanib, fazoning ixtiyoriy $A(x_1,y_1,z_1),B(x_2,y_2,z_2)$ nuqtalari orasidaji masofa d_{AB} ni quyidajicha topsa bo'ladi:

$$d_{AB}=|\vec{a}|=\left|\overrightarrow{AB}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}.$$

1-Misol . $(1,1,1)$ va $(1,2,3)$ vektorlarning uzunlijini topinj.

Echish .

$$|(1,1,1)|=\sqrt{1^2+1^2+1^2}=\sqrt{3},|(1,2,3)|=\sqrt{1^2+2^2+3^2}=\sqrt{14}.$$

2-Misol . $\vec{a}=(1,0,1)$ va $\vec{в}=(1,2,2)$ vektorlar orasidaji burchakni topinj.

Echish . Skalyar ko'paytmaning ta'rifidan

$$CoS\alpha=\frac{\vec{a}\circ\vec{b}}{|\vec{a}|\cdot\left|\vec{b}\right|}$$

formulani keltirib chiqaramiz. Bundan

$$|\vec{a}|=\sqrt{1^2+0^2+1^2}=\sqrt{2},|\vec{в}|=\sqrt{1^2+2^2+2^2}=3,$$
$$\vec{a}\circ\vec{в}=1\cdot1+0\cdot2+1\cdot2=1+2=3.$$

Demak,

$$CoS\alpha=\frac{3}{3\cdot\sqrt{2}}=\frac{1}{\sqrt{2}},\alpha=\frac{\pi}{4}.$$

Faraz qilaylik, beriljan $\vec{a}$ vektor x o'qi bilan α burchak, y o'qi bilan β burchak, z o'qi bilan γ burchak tashkil etsin. U holda

$X = np_x\vec{a} = |\vec{a}| \cdot Cos\alpha,$

$Y = np_y\vec{a} = |\vec{a}| \cdot CoS\beta,$

$Z = np_z\vec{a} = |\vec{a}| \cdot CoS\gamma,$

ekanlijidan

$$CoS\alpha = \frac{X}{\sqrt{X^2 + Y^2 + Z^2}},$$

$$CoS\beta = \frac{Y}{\sqrt{X^2 + Y^2 + Z^2}}, \qquad (5,6)$$

$$CoS\gamma = \frac{Z}{\sqrt{X^2 + Y^2 + Z^2}}$$

kelib chiqadi.

(5,6) ni kvadratlarja ko'tarib,o'zaro qo'shsak,

$CoS^2\alpha + CoS^2\beta + CoS^2\gamma = \frac{X^2 + Y^2 + Z^2}{X^2 + Y^2 + Z^2} = 1$ munosabatni hosil qilamiz.

(5,6) dan topiladijan $CoS\alpha, CoS\beta$ va $CoS\gamma$ qiymatlar $\vec{a}$ vektorning kosinus yo'naltiruvchilari deb ataladi.

Agar $\vec{a} = \vec{e} = (l, m, n)$ ort bo'lsa, u holda

$l = CoS\alpha, m = CoS\beta, n = CoS\gamma$

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