BA’ZI INTEGRAL OPERATORLARNING XOS QIYMATLARINI ANIQLASH

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Masalaning qo’yilishi. $\varphi_1(x),...,\varphi_m(x)$ va $\psi_1(s),...,\psi_m(s)$ uzliksiz funktsiyalar orqali aniqlangan A operatorning xos qiymatlarini ya’ni spektirini aniqlshning algoritmini tuzish va maple dasturidan foydalanib xos qiymatlarini tekshirish.

Aytaylik, chiziqli $L_2([a,b]^n)$ fazoda quyidagi A operator berilgan bo’lsin

$(Af)(x_1,x_2,\cdots,x_n)=\int_a^b\int_a^b\cdots\int_a^b K(x_1,x_2,\cdots,x_n;s_1,s_2,\cdots,s_n)f(s_1,s_2,\cdots,s_n)ds_1ds_2\cdots ds_n$ bu yerda $K(x_1,x_2,\cdots,x_n;s_1,s_2,\cdots,s_n)$ – $[a,b]^n\times[a,b]^n$, $a<b$ to’plamda aniqlangan uzluksiz funksiya yoki A operatorning yadrosi deb qarashimiz mumkin.

Quyidagi teoramani isbotsiz keltiramiz.

Teorema 1. $K(x_1,\cdots,x_n;s_1,\cdots,s_n)=\sum_{i=1}^{m}\phi_i(x_1,\cdots,x_n)\psi_i(s_1,\cdots,s_n)$, $m<\infty$ *bo’lsin.* $L_2([a,b]^n)$ *fazoda aniqlangan* A *operatorning qiymatlari to’plami chekli o’lchamli fazo bo’ladi, ya’ni* A *chekli operator bo’ladi.*

Shunday qilib, berilgan operator xos qiymatga ega bo’lishi uchun

$$\begin{cases}(\frac{1}{z}a_{11}-1)c_1+\frac{1}{z}a_{12}c_2+...+\frac{1}{z}a_{1m}c_m=0\\ \frac{1}{z}a_{21}c_1+(\frac{1}{z}a_{22}-1)c_2+...+\frac{1}{z}a_{2m}c_m=0\\ \dots\dots\dots\dots\dots\dots\dots\dots\\ \frac{1}{z}a_{m1}c_1+\frac{1}{z}a_{m2}c_2+...+(\frac{1}{z}a_{mm}-1)c_m=0\end{cases}$$

sistema noldan farqli echimga ega bo’lishi zarur va etarli. Bu yerda $a_{ij}=\int_a^b\cdots\int_a^b\psi_i(s_1,\cdots,s_n)\varphi_j(s_1,\cdots,s_n)ds_1ds_2\cdots ds_n$ va $c_i=\sum_{j=1}^{n}a_{ij}(z)c_j$, $i=1,2,...,n$.

Masalani yechish algoritmi

I-qadam. Operator ko’rinishini yozamiz

$$Af(x_1,\cdots,x_n)=\int_a^b\cdots\int_a^b\sum_{i=1}^{m}\varphi_i(x_1,\cdots,x_n)\psi_i(s_1,\cdots,s_n)f(s_1,\cdots,s_n)ds_1\cdots ds_n.$$

II-qadam. operatorning n, $\varphi_1(x_1,\cdots,x_n),...,\varphi_m(x_1,\cdots,x_n)$ va $\psi_1(s),...,\psi_n(s)$, a,b parametrlariga qiymatlar beramiz.

III-qadam. $a_{ij}=\int_a^b\cdots\int_a^b\psi_i(s_1,\cdots,s_n)\varphi_j(s_1,\cdots,s_n)ds_1ds_2\cdots ds_n$ sonlarni hisoblaymiz.

IV-qadam. $\Delta(z)=\begin{vmatrix} a_{11}-z & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22}-z & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mm}-z \end{vmatrix}$ determinantni analitik ko'rinishini topamiz, ya'ni bu determinantni hisoblab, Z ga nisbatan m-tartibli ko'phadni ko'rinishini aniqlaymiz.

V-qadam. $\Delta(z)$ nollari mavjudligi va mavjud bo'lsa ularni aniqlaymiz.

Misol.

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> with(student):
> restart;
> print("OPERATORNI YADROSINI KIRITING");
K(x,y)='Sum(phi[i](x)*Q[i](s),ii=1..n)';
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"OPERATORNI YADROSINI KIRITING" $K(x,y)=\sum_{ii=1}^{n}\phi_i(x)\,Q_i(s)$

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> n:=4;
> for l from 1 to n do
P[l](y):=l*cos(y1)*cos(y2);
Q[l](y):=l*cos(y1)*cos(y2);end;
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$P_1(y) := \cos(y1)\cos(y2)$ - $Q_1(y) := \cos(y1)\cos(y2)$ - $P_2(y) := 2\cos(y1)\cos(y2)$ - $Q_2(y) := 2\cos(y1)\cos(y2)$ - $P_3(y) := 3\cos(y1)\cos(y2)$ - $Q_3(y) := 3\cos(y1)\cos(y2)$

$P_4(y) := 4\cos(y1)\cos(y2)$ - $Q_4(y) := 4\cos(y1)\cos(y2)$

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> lambda*f(x1,x2)=Sum(phi[i](x1,x2)*Int(Int(Q[i](s1,s2)*f(s1,s2),s1),s2),ii=1..n);
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$$\lambda\, f(x1,x2)=\sum_{ii=1}^{4}\phi_i(x1,x2)\int\!\!\int Q_i(s1,s2)\,f(s1,s2)\,ds1\,ds2$$

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> for l from 1 to n do
P[l](y):=l*cos(y1)*cos(y2):
Q[l](y):=l*cos(y1)*cos(y2):end;
for l from 1 to n do
for k from 1 to n do
A[l,k]:=int(int(Q[l](y)*P[k](y)/lambda,y1=-1..1),y2=-1..1):
MM[l,k]:=evalf(A[l,k]);
end;end;
for l from 1 to n do
for k from 1 to n do
if (MM[l,k]=MM[k,k]) then BB[l,k]:=(1-MM[l,k])
else BB[l,k]:=-MM[l,k] end  if;
end;end;
 with(LinearAlgebra):
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M:=Matrix(1..n,1..n,BB);dett:=Determinant(M);

$P_1(y) := \cos(y1)\cos(y2)$ $Q_1(y) := \cos(y1)\cos(y2)$ $P_2(y) := 2\cos(y1)\cos(y2)$
$Q_2(y) := 2\cos(y1)\cos(y2)$ $P_3(y) := 3\cos(y1)\cos(y2)$ $Q_3(y) := 3\cos(y1)\cos(y2)$
$P_4(y) := 4\cos(y1)\cos(y2)$ $Q_4(y) := 4\cos(y1)\cos(y2)$

$$M := \begin{bmatrix} 1 - \frac{2.116002878}{\lambda} & -\frac{4.232005756}{\lambda} & -\frac{6.348008634}{\lambda} & -\frac{8.464011512}{\lambda} \\ -\frac{4.232005756}{\lambda} & 1 - \frac{8.464011512}{\lambda} & -\frac{12.69601727}{\lambda} & -\frac{16.92802302}{\lambda} \\ -\frac{6.348008634}{\lambda} & -\frac{12.69601727}{\lambda} & 1 - \frac{19.04402590}{\lambda} & -\frac{25.39203454}{\lambda} \\ -\frac{8.464011512}{\lambda} & -\frac{16.92802302}{\lambda} & -\frac{25.39203454}{\lambda} & 1 - \frac{33.85604605}{\lambda} \end{bmatrix}$$

$$dett := \frac{\lambda^4 - 63.48008634\ \lambda^3 - 0.22\ 10^{-6}\ \lambda^2 + 0.5924808072\ \ 10^{-6}\ \lambda - 0.19\ 10^{-14}}{\lambda^4}$$

> #determinantning nollarini topamiz, ya'ni
#K operatorning xosqiymatlarini topamiz
solve(dett=0, lambda);

-0.00009661244110, $0.3206854942\,10^{-8}$, 0.00009660591562, 63.48008634

> print("K operatorning xos qiymatlari 4 ta ekan:");
-.9661244110e-4;.3206854942e-8;.9660591562e-4;63.48008634;

"K operatorning xos qiymatlari 4 ta ekan:"

-0.00009661244110 $0.3206854942\,10^{-8}$ 0.00009660591562 63.48008634

> DD:=lambda^4-63.48008634*lambda^3-.22e-6*lambda^2+.5924808072e-6*lambda-.19e-14;

$$DD := \lambda^4 - 63.48008634\lambda^3 - 0.22\ 10^{-6}\ \lambda^2 + 0.5924808072\,10^{-6}\ \lambda - 0.19\ 10^{-14}$$

> plot(DD,lambda=-0.00010..0.00010);

ADABIYOTLAR:

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