HOSILA UNING FIZIK VA GEOMETRIK MA'NOSI

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Annotatsiya: *Ushbu maqolada hosilaning fizik va geometrik ma'nosini ko'rib chiqish, hosilani topish algoritmi, ushbu algoritm yordamida funksiyaning hosilasini hisoblashni o'rganish, matematik munosabatlarni topish jarayonida kuzatuvchanlikni rivojlantirish, matematikaga qiziqishni oshirish xususida so'z yuritilgan*

Kalit so'zlar: *hosila tushunchasi, fizik, matematik, xossa, tushuncha*

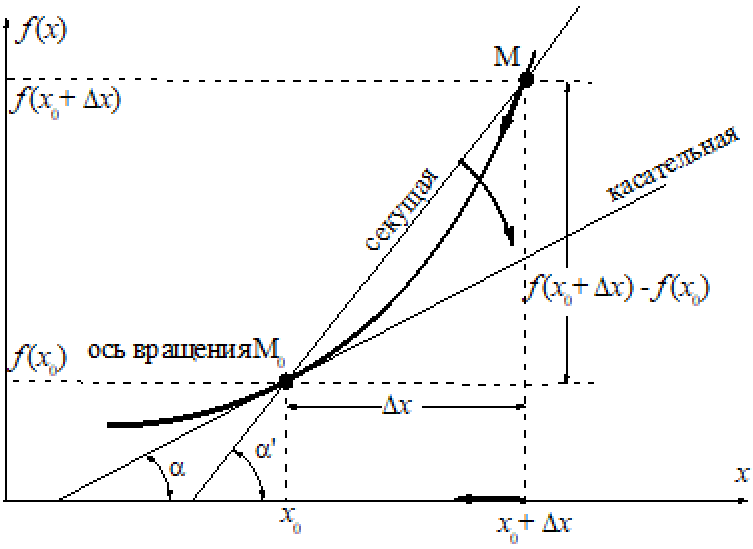
HOSILA TUSHUNCHASIGA OLIB KELADIGAN MUAMMOLAR

Muammo 1 (lahzali tezlik haqida). Ba'zi moddiy nuqta *x o'qi* bo'ylab harakatlansin , shunda *x (t)* nuqtaning *t vaqtidagi koordinatasi bo'lsin* . Vaqt o'tgach, Δt nuqta koordinatasi bo'ladi $x(t+\Delta t)$, ya'ni. vaqt o'tishi bilan Δt nuqta masofani bosib o'tadi $\Delta x = x(t+\Delta t) - x(t)$. Shuning uchun nuqtaning vaqt oralig'idagi o'rtacha tezligi Δt ga teng bo'ladi $\frac{\Delta x}{\Delta t}$. Vaqtning bir lahzasidagi nuqtaning oniy tezligini topish uchun nolga t moyil bo'lish kerak Δt, ya'ni

$$V(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

2-masala (grafikga tegish haqida). Egri chiziq tenglama bilan berilgan bo'lsin $y = f(x)$. Uning ikkita M_0 (x_0, $f(x_0)$) va M ($x_0+\Delta x$, $f(x_0+\Delta x)$) nuqtalarini sekant bilan bog'laymiz .

= $\mathrm{tg}\,\alpha'$ kasr $\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$, bu erda sekantning OX o'qiga moyillik burchagi (α' M_0MT uchburchakda oyoqlarning nisbati)



M nuqtada $\Delta x \to 0$ M_0 nuqtaga qarab harakatlana boshlaydi . Bunda butun sekant M_0 nuqta atrofida aylanadi va chegarada u M_0 nuqtaga tangensga aylanadi . Keyin burchak α' bu tangens *x o'qi bilan hosil qilgan* burchakka o'tadi α. Shuning uchun buni bahslash mumkin

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \operatorname{tg} \alpha = k,$$

Qayerda α –nuqtadagi egri chiziqqa x_0 va OX o'qiga tegib hosil qilgan burchak, k –- tangensning qiyaligi.

3-masala (kimyoviy reaksiya tezligi haqida) *(t) t vaqt ichida reaksiyaga kirgan moddaning miqdori bo'lsin . γVaqt o'tgach, Δt reaksiyaga kirishgan moddaning miqdori bo'ladi $\gamma(t+\Delta t)$, ya'ni. vaqt davomida Δt reaksiyaga kirgan moddaning miqdori $\Delta\gamma = \gamma(t+\Delta t) - \gamma(t)$. Shuning uchun vaqt oralig'idagi kimyoviy reaksiyaning o'rtacha tezligi Δt ga teng bo'ladi $\frac{\Delta\gamma}{\Delta t}$. Kimyoviy reaksiyaning bir lahzadagi tezligini topish uchun nolga t moyil bo'lish kerak Δt, ya'ni*

$$V(t) = \lim_{\Delta t \to 0} \frac{\gamma(t+\Delta t) - \gamma(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\gamma}{\Delta t}$$

Limit yordamida, $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ ko'rib chiqilganlardan tashqari, boshqa ko'plab muhim muammolar hal qilinadi (masalan: o'tkazgichda oqadigan o'zgaruvchan tokning kattaligi masalasi; bir hil bo'lmagan novda, issiqlikning chiziqli zichligini topish. qizdirilganda jismning sig'imi, aylanadigan jismning burchak tezligi va boshqalar), keyin bu chegarani har tomonlama o'rganish, xususan, uni hisoblash usullarini ko'rsatish tavsiya etiladi. Matematikada bu chegara **hosila deyiladi.**

1. <u>Hosila, uning geometrik va fizik ma'nosi</u>

Ta'rif: Berilgan *y=f(x) funksiyaning hosilasi D y funksiyaning o'sishining D x* argumentining o'sish qismiga nisbatining chegarasi, ikkinchisi nolga moyil bo'lganda va bunday chegara mavjud va chekli bo'ladi. .

x argumentining funktsiyasi sifatida ko'rib chiqilishi mumkin . Bu funksiya $f'(x)$ bilan belgilanadi.

Hosila $f'(x)$, y', belgilar bilan belgilanadi $\frac{dy}{dx}$. *X=a* da hosilaning xususiy qiymati $f'(a)$ yoki $y'|_{x=a}$.

f(x) funksiyaning hosilasini topish amali bu funksiyani differentsiallash deyiladi.

Ta'rif bo'yicha hosilani to'g'ridan-to'g'ri topish uchun quyidagi asosiy ***qoida qo'llanilishi mumkin :***

1. *X ga D x* o'sish qiymatini bering va f(x + Dx) funksiyaning o'sish qiymatini toping .
2. funksiyaning o'sish qismini toping .
3. Munosabat o'rnating $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$ va bu nisbatning D *x*⟶ *0* da chegarasini toping .

Misollar.

1. $y = x^2$ funksiyaning hosilasini toping

a) ixtiyoriy nuqtada;

$x = 2$ nuqtada .

A)

1. $f(x + Dx) = (x + Dx)^2$;

2. $Dy = (x + Dx)^2 - x^2 = 2xDx - x^2$;

3. $y' = \lim_{\Delta x \to 0} \frac{2x\Delta x - \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} (2x - \Delta x) = 2x$.

b) $f'(2) = 4$

2. Ta'rifdan foydalanib, funksiyaning $y = \sqrt{1+2x}$ ixtiyoriy nuqtadagi hosilasini toping.

1. $f(x+\Delta x) = \sqrt{1+2(x+\Delta x)}$.

2. $\Delta y = \sqrt{1+2(x+\Delta x)} - \sqrt{1+2x}$

3. $\lim_{\Delta x \to 0} \frac{\sqrt{1+2x+2\Delta x} - \sqrt{1+2x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x(\sqrt{1+2x+2\Delta x} + \sqrt{1+2x})} = \frac{1}{\sqrt{1+2x}}$.

Bir lahzali tezlik va grafikga tegish muammolarini hisobga olgan holda, quyidagi bayonotlarni tuzish oson:

<u>Hosilaning fizik ma'nosi:</u> *notekis harakat* tezligi vaqtga nisbatan bosib o'tgan masofaning hosilasidir.

$$V(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = X'(t)$$

Hosilaning geometrik ma'nosi: $y'(x_0)$ bu funksiyaning grafigiga x_0 *nuqtadagi tangensning burchak koeffitsientini ifodalaydi* (ya'ni, x argumentining berilgan qiymati uchun hosila tangensdan hosil bo'lgan burchakning tangensiga tengdir. $f(x)$ funksiyaning tegishli $M_0(x;y)$ nuqtasida Ox o'qining musbat yo'nalishi bilan grafigi *)*.

Ta'rif. M $_0$ (x_0 , y_0) nuqtadagi funktsiya **grafigiga teginish M** $_0$ M nuqtaning egri chiziq bo'ylab harakatlanayotganda M $_0$ nuqtaga to'g'ri kelishga intilayotganida sekantning chegaralanish holati M 0 M deb ataylik .

$_0$(x_0, y_0) nuqtadagi funksiya grafigiga teginish tenglamasi :

$y - y_0 = f'(x_0)(x - x_0)$.

(formulani o'zingiz chiqaring, y=kx+b-chiziq tenglamasi, k= $f'(x_0)$)

2. <u>Yig'indi/farqning hosilasi</u>

Shunday qilib, biz hosilani aniqladik va uning fizik va geometrik ma'nosini tushuntirdik. Endi biz keyingi qadamni qo'yishimiz kerak - farqlash qoidalarini ko'rib chiqing.

Limit yordamida hosilani topishning umumiy usulidan foydalanib, eng oddiy differensiallash formulalarini olish mumkin. $u=u(x)$,$v=v(x)$ x o'zgaruvchining ikkita differentsiallanuvchi funksiyasi bo'lsin .

Farqlashning asosiy qoidalari quyidagi formulalar bilan ifodalanadi:

1. $(u \pm v)' = u' \pm v'$

• $(u \pm c)' = u'$

2. $(u \cdot v)' = u' \cdot v + u \cdot v'$

• $(c \cdot u)' = c \cdot u'$

3. $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

• $\left(\frac{u}{c}\right)' = \frac{u'}{c}$

• $\left(\frac{c}{v}\right)' = -\frac{c \cdot v'}{v^2}$

1 va 2 formulalarni o'zingiz isbotlang.

<u>Formula 1 isboti</u> .

bo'lsin . X +D x argumentining qiymati uchun bizda y (x +D x)= u (x +D x) + v (x +D x) mavjud.

Keyin

D y = y (x +D x) – $y(x)$ = $u(x$ +D $x)$ + $v(x$ +D $x)$ – $u(x)$ – $v(x)$ = D u +D v .

Demak,

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} + \lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} = u' + v'.$$

(tergov xulosasini tuzing)

3. <u>Mahsulot/qismning hosilasi</u>

<u>2-formulaning isboti</u> .

bo'lsin . U holda y (x +D x)= u (x +D x) v (x +D x), shuning uchun

$\Delta y = u(x+\Delta x)v(x+\Delta x) - u(x)v(x)$.

u va *v* funksiyalarning har biri *x* nuqtada differensiallanar ekan , u holda ular shu nuqtada uzluksiz bo'ladi, ya'ni $u(x+\Delta x) \to u(x)$, $v(x+\Delta x) \to v(x)$, $\Delta x \to 0$ da.

Shuning uchun biz yozishimiz mumkin

$$y' = \lim_{\Delta x\to 0} \frac{u(x+\Delta x)\cdot v(x+\Delta x) - u(x)v(x)}{\Delta x} =$$

$$= \lim_{\Delta x\to 0} \frac{u(x+\Delta x)v(x+\Delta x) - u(x)v(x+\Delta x) + u(x)v(x+\Delta x) - u(x)v(x)}{\Delta x} =$$

$$= \lim_{\Delta x\to 0} v(x+\Delta x)\frac{u(x+\Delta x)}{\Delta x} + \lim_{\Delta x\to 0}\frac{v(x+\Delta x)}{\Delta x}u(x) = u'\cdot v + u\cdot v'.$$

Ushbu xususiyatga asoslanib, istalgan sonli funktsiyalar mahsulotini farqlash qoidasini olish mumkin.

Masalan, $y=u\cdot v\cdot w$ *bo'lsin*. Keyin,

$y' = u'\cdot(vw) + u\cdot(vw)' = u'\cdot vw + u\cdot(v'w + vw') = u'\cdot vw + u\cdot v'\cdot w + u\cdot v\cdot w'$.

(tergov xulosasini tuzing)

<u>3-formulaning isboti</u> .

Mayli $y = \dfrac{u(x)}{v(x)}$. Keyin

$$y(x+\Delta x) = \frac{u(x+\Delta x)}{v(x+\Delta x)}.$$

$$\Delta y = \frac{u(x+\Delta x)}{v(x+\Delta x)} - \frac{u(x)}{v(x)} = \frac{u(x+\Delta x)v(x) - v(x+\Delta x)u(x)}{v(x+\Delta x)v(x)}.$$

$$y = \lim_{\Delta x\to 0} \frac{u(x+\Delta x)v(x) - v(x+\Delta x)}{v(x+\Delta x)v(x)\Delta x} =$$

$$= \lim_{\Delta x\to 0} \frac{1}{v(x+\Delta x)v(x)}\cdot\frac{u(x+\Delta x)v(x) - u(x)v(x) + u(x)v(x) - v(x+\Delta x)u(x)}{\Delta x} =$$

$$= \frac{1}{v^2}\lim_{\Delta x\to 0}\left[\frac{u(x+\Delta x)}{\Delta x}v(x) - \frac{v(x+\Delta x)}{\Delta x}u(x)\right] = \frac{1}{v^2}(u'\cdot v - u\cdot v') = \frac{u'\cdot v - u\cdot v'}{v^2}.$$

Isbotda $v(x+\Delta x) \to v(x)$ ning $\Delta x \to 0$ ekanligidan foydalandik.

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