TRIGONOMETRIYA TARIXI

Yoqubjon Botirov Qosimovich
Toshkent davlat transport universiteti akademik litseyi
Matematika fani o'qituvchisi.

Annotatsiya: *Trigonometriya kelib chiqish tarixi, qadimgi hisob kitoblar izlanishlar va trigonometriya deb atalishi, Miloddan avvalgi III asrda Yevklid va Arximed kabi yunon matematiklari akkordlar va aylanalarga chizilgan burchaklarning xossalarini o'rganib, X asrga kelib islom matematiklari barcha oltita trigonometrik funksiyadan foydalanib, G'arbiy Yevropaga Ptolemeyning yunoncha "Almagest" asarining lotincha tarjimalari, XVI asrda Shimoliy Yevropada trigonometriya hali ham kam miqdorda ma'lum edi.*

Kalit so'zlar: *Trigonometriya, matematika, uchburchak, Yevklid, Arximed, Gipparx, Ptolomey, Nosiriddin at-Tusiy, Al Battani, Jeyms Gregori(XVII asr), Kolin Maklaurin(XVIII asr), Sinus, kosinus, tangens, kotangens, burchak, ayniyat, formulalar.*

Trigonometriya (yunonchadan "trigon" - uchburchak, "metrezis" - o'lchash so'zlaridan olingan bo'lib, o'zbek tiliga "uchburchaklarni o'chash" deya tarjima qilinadi) - matematikaning asosiy bo'limlaridan biri hisoblanib, uchburchak tomonlari va burchaklari orasidagi bog'lanishlar, trigonometrik funksiyalarning xossalari va ular o'rtasidagi bog'lanishlarni o'rganadi. Hindistonliklar ilk marta trigonometrik funksiyalar qiymatlari jadvalini kashf qilganlar. Shumer astronomlari aylanalarni 360 gradusga bo'lish orqali burchak o'lchovini o'rganishdi. Ular va keyinchalik bobilliklar o'xshash uchburchaklar tomonlari nisbatlarini o'rgandilar va bu nisbatlarning ba'zi xususiyatlarini kashf etdilar, lekin buni uchburchaklarning tomonlari va burchaklarini topishning tizimli usuliga aylantirmadilar.Qadimgi nubiyaliklar ham xuddi shunday usuldan foydalanganlar. Miloddan avvalgi III asrda Yevklid va Arximed kabi yunon matematiklari akkordlar va aylanalarga chizilgan burchaklarning xossalarini o'rganib, zamonaviy trigonometrik formulalarga ekvivalent bo'lgan teoremalarni isbotladilar, garchi ular ushbu formulalarnini algebraik jihatdan emas, balki geometrik jihatdan isbot qilgan bo'lsalar ham. Miloddan avvalgi 140-yilda Gipparx (Nikea, Kichik Osiyo) zamonaviy sinus qiymatlari jadvallariga o'xshash akkordlarning birinchi jadvallarini bergan va ulardan trigonometriya va sferik trigonometriya masalalarini yechishda foydalangan. Milodiy II asrda yunon-misr astronomi Ptolomey (Misrning Iskandariya shahridan) o'zining "Almagest" asarining 1-kitobi, 11-bobida batafsil trigonometrik jadvallarni (Ptolemeyning akkordlar jadvali) tuzgan. Ptolemey o'zining trigonometrik funksiyalarini aniqlash uchun akkord uzunligidan foydalangan, bu biz ishlatadigan sinus funksiyasidan ozgina farq qiladi. Biz sin(α) deb ataydigan qiymatning akkord uzunligini Ptolemey jadvalidagi kerakli burchak qiymati ikki barobarini (2α) aniqlash va keyin bu qiymatni ikkiga bo'lish orqali topish mumkin. Batafsilroq jadvallar yaratilgunga qadar asrlar o'tdi va Ptolemeyning risolasi keyingi 1200 yil davomida O'rta asr Vizantiyasi, Islom va keyinchalik G'arbiy Yevropa dunyolarida astronomiyada trigonometrik hisoblarni amalga oshirish uchun ishlatilgan. Zamonaviy sinus funksiyasi birinchi marta Surya Siddxantada uchragan va uning xususiyatlarini V asrda (milodiy) hind matematiki va astronomi Aryabhata hujjatlashtirgan. Bu yunon va hind asarlari o‘rta asr islom matematiklari tomonidan tarjima qilingan va kengaytirilgan. X asrga kelib islom matematiklari barcha oltita trigonometrik funksiyadan foydalanib, ularning qiymatlarini jadvalga kiritib, sferik geometriya masalalariga qo‘llaganlar. Fors olimi Nosiriddin at-Tusiy trigonometriyaning o‘ziga xos matematik fan sifatida yaratuvchisi sifatida ta’riflangan.

U birinchi bo'lib trigonometriyani astronomiyadan mustaqil matematik fan sifatida ko'rib chiqdi va sferik trigonometriyani hozirgi shaklga keltirdi. U sferik trigonometriyada to‘g‘ri burchakli uchburchakning oltita aniq holatlarini sanab o‘tdi va o‘zining “Sektor rasmi to‘g‘risida” asarida tekislik va sferik uchburchaklar uchun sinuslar qonunini bayon qildi, sferik uchburchaklar uchun tangenslar qonunini ochdi va ikkalasiga ham isbotlar keltirdi. Trigonometrik funksiyalar va usullar haqidagi bilimlar G'arbiy Yevropaga Ptolemeyning yunoncha "Almagest" asarining lotincha tarjimalari, shuningdek, Al Battani va Nosiriddin at-Tusiy kabi fors va arab astronomlarining asarlari orqali yetib bordi. Shimoliy yevropaliklarning matematikada trigonometriyaga oid eng qadimgi asarlaridan biri bu XV asr nemis matematigi Regiomontanusning "De Triangulis" asari bo‘lgan. Shu bilan birga, Almagestning yunon tilidan lotin tiliga yana bir tarjimasi Jorj Trebizond tomonidan yakunlandi. XVI asrda Shimoliy Yevropada trigonometriya hali ham kam miqdorda ma'lum edi. Navigatsiya talablari va yirik geografik hududlarning aniq xaritalariga ortib borayotgan ehtiyoj tufayli trigonometriya matematikaning asosiy sohasiga aylandi. Trigonometriya so'zi ilk bor Bartholomeush Pitiushning 1595-yilda chop etilgan "Trigonometriya" asarida uchragan. Kompleks sonlarni trigonometriyaga to‘liq kiritgan shved olimi Leonard Eyler edi. Shotland matematiklari Jeyms Gregori(XVII asr) va Kolin Maklaurin(XVIII asr)ning ishlari trigonometrik qatorlarning rivojlanishiga ta’sir ko‘rsatdi. Yana, XVIII asrda Bruk Teylorning Teylor seriyalari yaralgan. Ushbu ma’lumotlar orqali siz trigonometriyaning kelib chiqish tarixini bilib olishingiz mumkin, biz matematiklar judayam ko’p foydalanadigan “Pifagor” teoremalari va formulalari isboti barchaga ma’lum albatta.

Sinus, kosinus, tangens va kotangensning ta'riflari birinchi navbatda geometriyada to'g'ri burchakli uchburchak tomonlari munosabati orqali berilgan.

To'g'ri burchakli uchburchakdagi o'tkir burchak - bu qarama-qarshi oyoqning gipotenuzaga nisbati.

kosinus to'g'ri burchakli uchburchakdagi o'tkir burchak - qo'shni oyoqning gipotenuzaga nisbati.

tangens to'g'ri burchakli uchburchakdagi o'tkir burchak - bu qarama-qarshi oyoqning qo'shnisiga nisbati.

Kotangent to'g'ri burchakli uchburchakdagi o'tkir burchak qo'shni oyoqning teskarisiga nisbati deb ataladi.

Ushbu ta'riflar faqat o'tkir burchaklar uchun amal qiladi (0° dan 90 ° gacha).

FORMULALAR

1. Triganametrik ayniyatlar.

1) $\sin^2\alpha+\cos^2\alpha=1$

2) $tg\alpha\cdot ctg\alpha=1$

3) $1+tg^2\alpha=\frac{1}{\cos\alpha}$

4) $1+ctg^2\alpha=\frac{1}{\sin\alpha}$

5) $tg\alpha=\frac{\sin\alpha}{\cos\alpha}$

6) $ctg\alpha=\frac{\cos\alpha}{\sin\alpha}$

2. Trigonometrik funksiyalarni birinchisini ikkinchisi orqali ifodalash.

1) a) $\sin\alpha=\pm\sqrt{1-\cos^2\alpha}$

b) $\sin\alpha=\pm\frac{tg\alpha}{\sqrt{1+tg\alpha}}$

c) $\sin\alpha=\frac{1}{\sqrt{1+ctg\alpha}}$

2) a) $\cos\alpha=\pm\sqrt{1-\sin^2\alpha}$

b) $\cos\alpha=\pm\frac{1}{\sqrt{1+tg\alpha}}$

c) $\cos\alpha=\pm\frac{1}{\sqrt{1+tg\alpha}}$

3) a) $tg\alpha=\pm\frac{1}{\sqrt{1-\sin\alpha}}$

b) $tg\alpha=\pm\frac{\sqrt{1-\cos\alpha}}{\cos\alpha}$

c) $tg\alpha=\frac{1}{ctg\alpha}$

4) a) $ctg\alpha=\pm\frac{\sqrt{1-\sin\alpha}}{\sin\alpha}$

b) $ctg\alpha=\pm\frac{\cos\alpha}{\sqrt{1-\cos\alpha}}$

c) $ctg\alpha=\frac{1}{tg\alpha}$

3. Trigonometrik funksiyalarni yig'indisi va ayirmasi.

1) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

2) $\sin(\alpha-\beta)=\sin\alpha\cos\beta - \cos\alpha\sin\beta$
3) $\cos(\alpha+\beta)=\cos\alpha\cos\beta - \sin\alpha\sin\beta$
4) $\cos(\alpha-\beta)=\cos\alpha\cos\beta + \sin\alpha\sin\beta$
5) $tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$
6) $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}$
7) $ctg(\alpha+\beta)=\dfrac{ctg\alpha ctg\beta-1}{ctg\alpha+ctg\beta}$
8) $ctg(\alpha-\beta)=\dfrac{ctg\alpha ctg\beta+1}{ctg\alpha-ctg\beta}$

1) a) $\sin\alpha=2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}$

b) $\sin2\alpha=2\sin\alpha\cos\alpha$

2) a) $\begin{cases}\cos\alpha=\cos^2\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}\\ \cos\alpha=1-2\sin^2\dfrac{\alpha}{2}\\ \cos\alpha=2\cos^2\dfrac{\alpha}{2}-1\end{cases}$

b) $\begin{cases}\cos\alpha=\cos^2\alpha-\sin^2\alpha\\ \cos\alpha=1-2\sin^2\alpha\\ \cos\alpha=2\cos^2\alpha\end{cases}$

3) $tg^2\alpha=\dfrac{2tg\alpha}{1-tg\alpha}$

4) $ctg2\alpha=\dfrac{1-tg\alpha}{2tg\alpha}$

5. Yarim burchak.

1) $\sin\dfrac{\alpha}{2}=\pm\dfrac{\sqrt{1-\cos\alpha}}{2}$

2) $\cos\dfrac{\alpha}{2}=\pm\dfrac{\sqrt{1-\cos\alpha}}{2}$

3) $tg\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{\sin\alpha}=\dfrac{1+\cos\alpha}{\sin\alpha}=\pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

4) $ctg\dfrac{\alpha}{2}=\dfrac{\sin\alpha}{1-\cos\alpha}=\dfrac{1+\cos\alpha}{\sin\alpha}=\pm\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}$

6. Darajani pasaytirish formulasi.

1) $\sin^2\alpha=\dfrac{1+\cos2\alpha}{2}$

2) $\cos^2\alpha=\dfrac{1-\cos2\alpha}{2}$

3) $\sin^6\alpha+\cos^6\alpha=\dfrac{5}{8}+\dfrac{3}{8}\cos^4\alpha$

7. Trigonometrik funksiyani ko'paytmadan yig'indiga aylantirish.

1) $\sin\alpha\cdot\cos\alpha=\dfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

2) $\cos\alpha\cdot\cos\beta=\dfrac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$

3) $\sin\alpha\cdot\sin\beta=\dfrac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

8. Triganametrik funksiyani yig'indidan ko'paytmaga aylantirish.

1) $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

2) $\sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

3) $\cos\alpha+\cos\beta=-2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

4) $\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

2) sin(α-β)=sinαcos β - cosαsin β
3) cos(α+β)=cosαcos β - sinαsin β
4) cos(α-β)= cosαcos β + sinαsin β
5) $tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$
6) $tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}$
7) $ctg(\alpha+\beta)=\frac{ctg\alpha ctg\beta-1}{ctg\alpha+ctg\beta}$
8) $ctg(\alpha-\beta)=\frac{ctg\alpha ctg\beta+1}{ctg\alpha-ctg\beta}$

1) a) $\sin\alpha=2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

b) sin2α=2sinαcosα

2) a) $\begin{cases}\cos\alpha=\cos^2\frac{\alpha}{2}-\sin\frac{\alpha}{2}\\ \cos\alpha=1-2\sin^2\frac{\alpha}{2}\\ \cos\alpha=2\cos^2\frac{\alpha}{2}-1\end{cases}$

b) $\begin{cases}\cos\alpha=\cos^2\alpha-\sin^2\alpha\\ \cos\alpha=1-2\sin^2\alpha\\ \cos\alpha=2\cos^2\alpha\end{cases}$

3) $tg^2\alpha=\frac{2tg\alpha}{1-tg\alpha}$

4) $ctg2\alpha=\frac{1-tg\alpha}{2tg\alpha}$

5. Yarim burchak.

1) $\sin\frac{\alpha}{2}=\pm\frac{\sqrt{1-\cos\alpha}}{2}$

2) $\cos\frac{\alpha}{2}=\pm\frac{\sqrt{1-\cos\alpha}}{2}$

3) $tg\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}=\frac{1+\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

4) $ctg\frac{\alpha}{2}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$

6. Darajani pasaytirish formulasi.

1) $\sin^2\alpha=\frac{1+\cos2\alpha}{2}$

2) $\cos^2\alpha=\frac{1-\cos2\alpha}{2}$

3) $\sin^6\alpha+\cos^6\alpha=\frac{5}{8}+\frac{3}{8}\cos^4\alpha$

7. Trigonometrik funksiyani ko'paytmadan yig'indiga aylantirish.

1) $\sin\alpha\cdot\cos\alpha=\frac{1}{2}$ [sin(α+β)+ sin(α-β)]

2) $\cos\alpha\cdot\cos\beta=\frac{1}{2}$ [cos(α+β)-cos(α-β)]

3) $\sin\alpha\cdot\sin\beta=\frac{1}{2}$ [cos(α-β)-cos(α+β)]

8. Triganametrik funksiyani yig'indidan ko'paytmaga aylantirish.

1) $\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

2) $\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

3) $\cos\alpha+\cos\beta=-2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

4) $\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$