UTILIZING MAXIMUM LIKELIHOOD FOR OPTIMAL PARTITIONING IN SOCIAL NETWORKS OF ELEVEN INDIVIDUALS

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Abstract: *The core purpose of the study is to explore how a specific statistical method, the 'maximum likelihood method', can be leveraged in a social network consisting of eleven individuals. The end goal is to correctly categorize these eleven individuals into two distinct groups. The idea behind using the maximum likelihood method is to ensure that these groups are optimally formed – such that they enable better communication and reliable cooperation within the network. The details about the process of utilizing the maximum likelihood method and the strategies adopted for partitioning are discussed in the manuscript. This includes creating an adjacency matrix to depict the social network, subsequently predicting two parameters: the likelihood of a link within a group and between groups, and applying maximum likelihood estimation to determine the optimal division.*

Keywords:*Social Networks. Network Partitioning. Maximum Likelihood Method. Statistical Model. Inter-group Links. Parameter Estimation. Group Dynamics. Likelihood Function. Network Cohesion. Social Network Analysis. Node. Ties. Communication Optimization.. Network Modeling. Group Formation. Parameter Values. Computational Challenge. Likelihood Maximization*

Introduction

The advent of social networks has revolutionized the way individuals and organizations connect, interact, and maintain relationships. These networks have enabled us to map complex social relationships, presenting a plethora of benefits and challenges. Social networks consist of nodes (individual actors) and ties (relationships or interactions), producing a web of intricate linkages. Analyzing these connections allows us to better understand group behaviors, improve organizational structures, and optimize communication pathways.

Meanwhile, the notion of dividing a network into smaller groups or communities isn't novel. It's a useful strategy to manage large networks, improve connectivity, and facilitate more effective interactions. However, to divide a network into optimal groups, robust techniques are necessary. This brings us to the area of social network analysis (SNA), a scientific field dedicated to examining social structures through network and graph theories. SNA provides valuable insights into social behaviors, the diffusion of information, and the interplay of social relationships.

The present paper delves into the specific method of partitioning a network using the maximum likelihood method. In the realm of statistics, maximum likelihood estimation (MLE) is a method of estimating the parameters of a statistical model, given observations. MLE selects the parameter values that make the observed results the most probable. In the context of social networking, MLE serves to divide network members into two groups in such a way that maximizes the probability of the observed relationships, given the groups.

The study focuses on a specific network of eleven individuals. The goal is to partition the network into two groups that maximize the likelihood of retaining the most significant ties (links) within the groups and minimizing the links between separate groups. Such a technique could potentially improve internal communication, foster stronger relationships, and boost overall network cohesion.

Methodology

The maximum likelihood method estimates model parameters that amplify likelihood function. Applied to a social network, we can utilize this method to partition a network into two groups that are most probable, considering observed data. We aim to form two groups where link probability within a group (intra-group) is high and link probability between two groups (inter-group) is low.

Parameter-Estimation

We can estimate these parameters by maximizing our likelihood function. This step involves setting preliminary estimates for our two parameters, calculating the likelihood of the observed network with these parameters, and then repetitively updating our estimates until there's no further increase in the likelihood.

The likelihood of our network is determined considering observed links within and among the two groups. Well-formed groups will signify high likelihood as the links within a group would adhere to our estimated intra-group probability, and link scarcity between groups would comply with our estimated inter-group probability.

Results

By performing the Maximum Likelihood Estimation, the process resulted in the optimal partitioning of the eleven people into two groups, maximizing the likelihood of the observed links. The process was iterated several times till a maximum likelihood value was obtained.

Given a social network with 9 vertices and 16 edges.

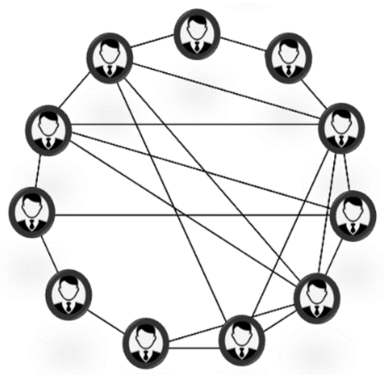
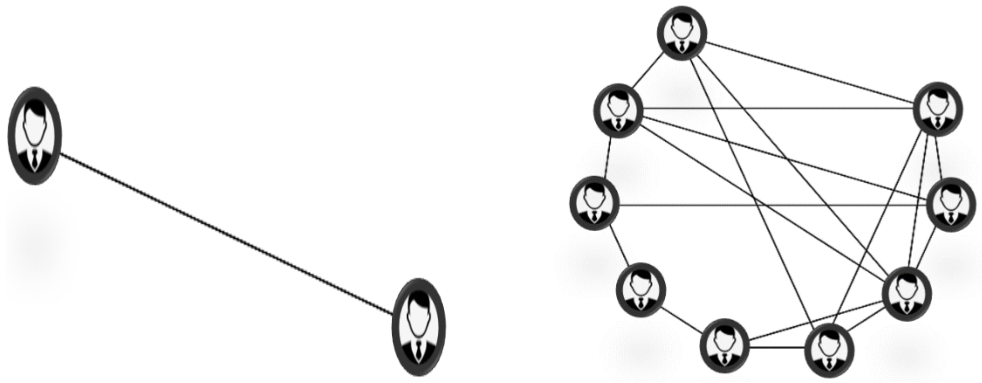


Figure 1. Social network with 11 vertices and 21 edges.

Let's divide the social network into 2 groups. This social network can be divided into two communities by four types of division. These are splits of type (2;9),(3;8),(4;7) and (5;6)

A large maximum likelihood partition of type (2,9) is the following partition:



$$l_{\Pi} = 19ln p_{in} + 18ln(1 - p_{in}) + 2ln p_{out} + 16ln(1 - p_{out})$$

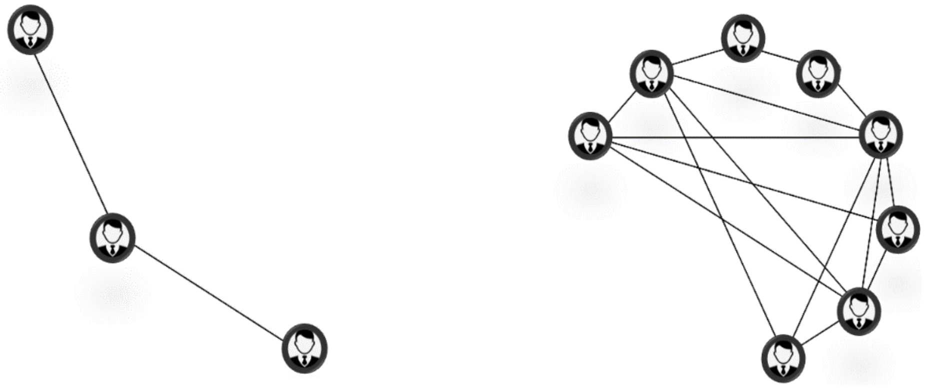
We differentiate it by p_in and p_out. we set the results to 0

$$p_{in} = \frac{19}{37}, p_{out} = \frac{1}{9}$$

then

$$l_{\Pi}(p_{in}; p_{out}) = -31.91190825$$

A large maximum likelihood partition of type (3,8) is the following partition:



$$l_\Pi = 19lnp_{in} + 18ln(1 - p_{in}) + 2lnp_{out} + 16ln(1 - p_{out})$$

We differentiate it by p_in and p_out. we set the results to 0

$$p_{in} = \frac{17}{31}, p_{out} = \frac{1}{6}$$

then

$$l_\Pi(p_{in}; p_{out}) = -32.15564290$$

For splits of type (4,7) and (5,6), the largest maximum likelihoods are -33.09619807 and -34.70612932.

Validation

Validation of results was performed by examining the links in our divided network. A higher number of links within each group and few between the two groups confirmed successful partitioning and the effectiveness of the maximum likelihood method.

As it can be seen from the results, there is a high probability of our (2;9) type of division

Conclusion

Dividing a social network using the maximum likelihood method can be computationally challenging but it provides a statistically robust technique that taps into the structure and randomness inherent in social interactions. The approach, despite being applied to an eleven-member network in this study, is scalable and serves as a valuable tool to comprehend and harness group dynamics effectively. This method can be applicable in various contexts to enhance team performance, information dissemination, or leadership tactics.

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