EXAMPLES ON THE CONSTRUCTION OF THE THE FUNCTION GREEN FOR n-ORDER HOMOGENEOUS DIFFERENTIAL EQUATIONS WITH BOUNDARY CONDITIONS

Sayliyeva Gulrukh Rustam kizi
Bukhara State University, teacher of the faculty of physics and mathematics
Jorayev Oybek Jasurbek ogli
Bukhara State University, student of the faculty of physics and mathematics

Anotation. *In this article, the existence condition of the function Green for n-order homogeneous differential equations satisfying the given boundary conditions is given, and the solution of several examples of the construction of the function Green is explained.*

Keywords: *the function Grin, boundary value problem, continuous function, function with discontinuity of type 1, jump of function at a point, linear form, linear arbitrary system.*

CHEGARAVIY SHARTLARGA EGA n-TARTIBLI BIR JINSLI DIFFERENSIAL TENGLAMALAR UCHUN GRIN FUNKSIYASINI QURISHGA DOIR MISOLLAR

Sayliyeva Gulrux Rustam qizi
Buxoro davlat universiteti, Fizika-matematika fakulteti o‘qituvchisi
Jo‘rayev Oybek Jasurbek o‘g‘li
Buxoro davlat universiteti, Fizika-matematika fakulteti talabasi

Annotatsiya. *Ushbu maqolada berilgan chegaraviy shartlarni qanoatlantiruvchi n-tartibli bir jinsli differensial tenglamalar uchun Grin funksiyasining mavjudlik sharti keltirilgan hamda Grin funksiyasini tuzishga doir bir necha xil misollarni yechish tushuntirilgan.*

Kalit so`zlar: *Grin funksiyasi, chegaraviy masala, uzluksiz funksiya, 1-tur uzilishga ega funksiya, funksiyaning biror nuqtadagi sakrashi, chiziqli forma, chiziqli erkli sistema.*

This article explains the function Green and its construction procedure for n-order homogeneous differential equations with boundary conditions. The function

Green is an important tool for solving differential equations by reducing them to integral equations.

Let us be given an n-order homogeneous differential equation of the following form

$$L[y] \equiv \rho_0(x)y^{(n)} + \rho_1(x)y^{(n-1)} + \cdots + \rho_n(x)y = 0 \qquad (1)$$

Coefficients $\rho_0(x), \rho_1(x), \dots, \rho_n(x)$ in equation (1) are infinite continuous functions on the section $[a, b]$. Equation (1) has the following boundary conditions:

$$V_k(y) = \alpha_k y(a) + \alpha_k{}^{(1)} y`(a) + \cdots + \alpha_k{}^{(n-1)} y^{(n-1)}(a) + \beta_k y(b) + \beta_k{}^{(1)} y`(b) + \cdots$$
$$+ \beta_k{}^{(n-1)} y^{(n-1)}(b), \qquad k = \overline{1, n}. \qquad (2)$$

In this case, the equations $V_1, V_2, \dots, V_n$ form a linear form. In them

$$y(a), y`(a), \dots, y^{(n-1)}(a);$$
$$y(b), y`(b), \dots, y^{(n-1)}(b)$$

systems form a linear system. We suppose that the boundary value problem (1)-(2) has only a trival solution.

Definition 1. [1] The function Green (effect function) constructed for the boundary value problem (1)-(2) is called $G(x, \xi)$ and is suitable for $\forall \xi \in (a, b)$ and has the following properties:

$1°$. The function $G(x, \xi)$ is continuous at $x \in [a, b]$, and has a continuous derivative up to $(n-2)$ order with respect to the variable x;

$2°$. The $(n-1)$ derivative of this function with respect to the variable x has a type 1 discontinuity at the point $x = \xi$, and the jump at this point is equal to the value (12)

$$\left.\frac{\partial^{n-1} G(x, \xi)}{\partial x^{n-1}}\right|_{x=\xi+0} - \left.\frac{\partial^{n-1} G(x, \xi)}{\partial x^{n-1}}\right|_{x=\xi+0} = \frac{1}{\rho_0(\xi)} \qquad (3)$$

and the jump at this point is equal to the value (3);

$3°$. In each of the intervals $[a, \xi]$ and $[\xi, b]$, $G(x, \xi)$ satisfies this equality $L[G]$;

$4°$. The function $G(x, \xi)$ satisfies (2)-boundary conditions

$$V_k(G) = 0, \quad k = \overline{1, n} \qquad (4)$$

Theorem 1. [1] If the boundary value problems (1) and (2) have only a trival solution $y(x) \equiv 0$, then the function $G(x, \xi)$ for the operator L exists and is unique.

Now let's see examples of the function Green construction. First, we will give an example of the simplest type. We are given the following boundary value problem.

Example 1.

$$y``` = 0; \quad y(0) = y`(0) = 0, \qquad y`(0) = y(1)$$

The simplicity of our examples is that its leading coefficient is 1 and the remaining coefficients are 0. Always, before solving the problem, we determine whether it is possible to construct a function Green for this boundary value problem. For this, we use Theorem 1 above. According to the condition of the theorem, the given higher-order homogeneous equation must have only a trival solution under the given boundary conditions. If this equation has at least one nonzero solution under the given boundary conditions, then it is impossible to construct a function Green for this differential equation. Initially

$$y^{```} = 0$$

solve the equation and determine its general solution. We create a solution of the following form:

$$y = C_1 \frac{x^2}{2} + C_2 x + C_3$$

Now, using the boundary conditions, we determine the unknown coefficients of the solution. From the boundary conditions the relations

$$C_1 = C$$

$$C_2 = -\frac{3}{2} C_1 = -\frac{3}{2} C$$

$$C_3 = -\frac{1}{2} C_1 = -\frac{1}{2}$$

are formed and it follows that the differential equation has infinitely many solutions.

Therefore, it is impossible to construct a function Green for this equation. Now let's look at the following example:

Example 2.

$$y^{(4)} = 0; \qquad y(0) = y^{`}(0) = y^{``}(0) = y^{```}(0) = 0.$$

Generated

$$y = \frac{C_1}{6} x^3 + \frac{C_2}{2} x^2 + C_3 x + C_4$$

by imposing boundary conditions on the general solution, we show that the differential equation has only zero solutions.So it is possible to construct the the function Green for the given differential equation with these boundary conditions. First, we construct a linearly independent system that constitutes the general solution. Since the function Green satisfies the boundary problem, its appearance must be the same as the solution of the differential equation.

$$y_1(x) = 1, \; y_2(x) = x, \quad y_3(x) = x^2, \quad y_3(x) = x^3 \qquad (5)$$

System (5) is a linear independent system. Now, using this general solution view, we can construct the function Green for the sections $0 \le x \le \xi$ and $\xi \le x \le 1$ in the following form:

$$G(x,\xi) = a_1 \cdot 1 + a_2 x + a_3 x^2 + a_4 x^3, \quad agar\ 0 \le x \le \xi\ bo'lsa \qquad (6)$$

$$G(x,\xi) = b_1 \cdot 1 + b_2 x + b_3 x^2 + b_4 x^3, \quad agar\ \xi \le x \le 11\ bo'lsa \qquad (7)$$

In equations (6) and (7), the coefficients $a_1,\ a_2,\ a_3, a_4,\ b_1,\ b_2,\ b_3, b_4$ are a continuous function of the variable ξ. For convenience, we introduce this notation $c_k = b_k - a_k$, and subtract the values of the expressions (6) and (7) themselves and their particular derivatives up to the $(n-1)$ th order with respect to the variable x at the point $x = \xi$. According to the main property of the the function Green $G(x,\xi)$, the eigenderivatives of this function up to the $(n-2)$ th order with respect to the variable x are continuous at the point $x = \xi$, but the $(n-1)$ –order derivative is at the point $x = \xi$ 1-type has a break.

The jump at this point

$$\left.\frac{\partial^{n-1} G(x,\xi)}{\partial x^{n-1}}\right|_{x=\xi+0} - \left.\frac{\partial^{n-1} G(x,\xi)}{\partial x^{n-1}}\right|_{x=\xi+0} = \frac{1}{\rho_0(\xi)} \qquad (8)$$

In equation (8), the function $\rho_0(x)$ is the leading coefficient of the n –order homogeneous differential equation. By performing the above operations, we create the following system of equations:

$$\begin{cases} c_1 + c_2 x + c_3 x^2 + c_4 x^3 = 0 \\ c_2 + 2c_3 x + 3c_4 x^2 = 0 \\ 2c_3 + 6c_4 x = 0 \\ 6c_4 = \dfrac{1}{\rho_0(x)} \end{cases} \qquad (9)$$

The main coefficient of the boundary problem we are looking at is $\rho_0(x) = 1$. The resulting system of equations (9) at $x = \xi$ forms a system of unknown linear equations $c_k,\ k = \overline{1,4}$.

$$\begin{cases} c_1 + c_2 \xi + c_3 \xi^2 + c_4 \xi^3 = 0 \\ c_2 + 2c_3 \xi + 3c_4 \xi^2 = 0 \\ 2c_3 + 6c_4 \xi = 0 \\ 6c_4 = 1 \end{cases} \qquad (10)$$

By solving the system of equations (10), we find the unknowns $c_k,\ k = \overline{1,4}$ depending on the variable ξ.

$$\begin{cases} c_1 = \frac{5}{6}\xi^3 \\ c_2 = \frac{1}{2}\xi^2 \\ c_3 = -\frac{1}{2}\xi \\ c_4 = \frac{1}{6} \end{cases} \qquad \begin{cases} b_1 - a_1 = \frac{5}{6}\xi^3 \\ b_2 - a_2 = \frac{1}{2}\xi^2 \\ b_3 - a_3 = -\frac{1}{2}\xi \\ b_4 - a_4 = \frac{1}{6} \end{cases}$$

We created 4 equations related to the unknown coefficients of the function Green. We extend the resulting system with new equations derived by applying the boundary conditions to the function Green. In this case, we apply the values of the boundary conditions at the lower point of the segment $[a, b]$ to the first (6) expression of the function Green, and the values of the upper point to the second (7) expression. As a result, we create the following equations related to the unknown coefficients of the function Green:

$$\begin{cases} a_1 = a_2 = 0 \\ 2\, b_3 + 6b_4 = 6b_4 = 0 \end{cases}$$

We solve the newly formed system of equations:

$$\begin{cases} b_1 - a_1 = \frac{5}{6}\xi^3 \\ b_2 - a_2 = \frac{1}{2}\xi^2 \\ b_3 - a_3 = -\frac{1}{2}\xi \\ b_4 - a_4 = \frac{1}{6} \\ a_1 = a_2 = 0 \\ 2\, b_3 + 6b_4 = 6b_4 = 0 \end{cases} \tag{11}$$

By solving the system (11), we determine all the unknown coefficients of the function Green and form the function Green as follows.

$$G(x,\xi) = \begin{cases} \frac{\xi}{2}x^2 - \frac{1}{6}x^3, & agar\ \ 0 \le x \le \xi\ \ bo'lsa \\ \frac{5}{6}\xi^3 + \frac{\xi^2}{2}x, & agar\ \ \xi \le x \le 11\ \ bo'lsa \end{cases}$$

The function Green also be constructed in the above manner for the following boundary value problems:

1) $y^{``} = 0; \quad y(0) = y^{`}(0), \qquad y^{`}(0) = y(1)$

2) $y^{``} = 0; \quad y(0) = y(1), \qquad y^{`}(0) = y^{`}(1)$

3) $y'' + y = 0; \quad y(0) = y(\pi) = 0,$

4) $y^{(4)} = 0; \quad y(0) = y'(0) = y''(1) = y'''(1) = 0$

5) $y''' = 0; \quad y(0) = y'(0) = 0, \qquad y'(0) = y(1)$

6) $y''' = 0; \quad y(0) = y(1) = 0, \qquad y'(0) = y'(1)$

7) $y'' = 0; \quad y(0) = 0, \qquad y'(1) = y(1)$

8) $y'' + y' = 0; \quad y(0) = y(1) = 0, \qquad y'(0) = y'(1)$

9) $y'' - k^2 y = 0; \ (k \neq 0), \qquad y(0) = y(1) = 0$

10) $y'' + y = 0; \quad y(0) = y(1), \qquad y'(0) = y'(1)$

11) $y''' = 0; \quad y(0) = y(1), \qquad y'(0) = y'(1)$

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