DEVELOPMENT OF MULTIDIMENSIONAL SIGNAL PROCESSING ALGORITHMS AND METHODS BASED ON BASIS FUNCTIONS

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Abstract: *Multidimensional signal processing algorithms and methods based on basis functions involve decomposing a signal into a set of basis functions that form a complete representation of the signal. This response provides an overview of the development of multidimensional signal processing algorithms and methods based on basis functions. The process involves selecting a set of orthogonal or linearly independent basis functions, representing the signal as a linear combination of these basis functions, applying signal analysis and processing techniques, and optimizing the algorithms for efficiency. This area of research has important applications in fields such as image and video processing, signal compression, and machine learning.*

INTRODUCTION

The advantage of the theory of spline functions with one variable is the good convergence of splines to approximated objects and the simplicity of the implementation of algorithms for constructing splines on a computer. The mathematical model of splines, developed over the past decades with the efforts of many researchers, has taken a worthy place among the methods and algorithms for approximation of functions. Karakoc et al. [1] introduced a lumped Galerkin finite element technique applying cubic B-spline basis functions for the BBM–Burgers equation. The aspects of Galerkin finite element method for the spatial approximation and precision analysis have been presented and discussed. Talat et al

The development of such algorithms and methods can be divided into several steps:

1. Basis function selection: The first step is to select a set of basis functions that will be used to represent the signal. These basis functions should be orthogonal or linearly independent, and they should span the entire signal space. Some common choices for basis functions include Fourier basis functions, wavelets, and polynomial basis functions.
2. Signal representation: Once the basis functions have been selected, the signal can be represented as a linear combination of these basis functions. This involves finding the coefficients that correspond to each basis function in the signal. This representation can be done in the time domain or frequency domain, depending on the choice of basis functions.
3. Signal analysis: After the signal has been represented in terms of the basis functions, various signal analysis techniques can be applied to the signal. For

example, the power spectrum of the signal can be computed by taking the Fourier transform of the signal representation.

4. Signal processing: The signal representation can also be manipulated in various ways to perform signal processing operations. For example, filtering can be done by selectively removing or attenuating certain frequency components of the signal representation.

5. Algorithm optimization: Finally, the performance of the signal processing algorithms can be optimized by choosing the most efficient basis functions, selecting the appropriate signal analysis and processing techniques, and optimizing the computational resources needed for the algorithms.

Overall, the development of multidimensional signal processing algorithms and methods based on basis functions is an important area of research with applications in many fields, including image and video processing, signal compression, and machine learning.

SIGNAL APPROXIMATION METHODOLOGY

One of the key characteristics of the spline is that the automatic analysis of signals using splines function which is carried out by multiplying the signal values and calculating their sum. It helps to parallelize computing processes. The usage of a cubic spline offers better performance than other approaches for parallel signal processing in real-time. A cubic interpolation spline is usually used as an initial approximation. The task of interpolating the function f(x) by a set of basis splines (B-spline) is reduced to finding the coefficients bi with the equation 1 taking such values that equation 2 is satisfied on the interval [a, b].

$$f(x) \sim= S_m = \sum_{i=-1}^{n+1} b_i(f)B_{m,i}(x) \qquad 1)$$

$$\sum_{i=0}^{n} b_i B_{m,i}(x_i) = f(x_i) \qquad 2)$$

The interpolation coefficients of functions are quite simply determined by B-splines of degree zero and first. They represent the heights of the interpolating rectangles and triangles at the reference points of the functions.

The problems of constructing interpolating and smoothing functions based on B-splines of higher degrees are solved in a more complex way. As a rule, the problem of finding b - coefficients implemented with solving a system of linear algebraic equations.

Various methods can be used to calculate the coefficients, such as local and interpolation equations, local compression, the least-squares method, and others. In this case, the approximation coefficients of B-splines are calculated using the least-squares method. In this case, it is necessary to compose and solve the quasi-diagonal system of linear algebraic equations

$$\sum_{k=1}^{n}(B_i, B_k)b_k) = (B_k),\ i = 1, 2, \ldots, n \quad (3)$$

where (B_i,B_k) (B_k,f) - scalar products.

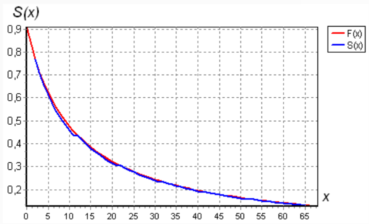
This system of equations is applicable for determining the bi coefficients within a local region, at its borders and beyond in close vicinity of the region, additional information should be used by introducing additional points

$i = -1, -2, \ldots, -m, n + 1, n + 2, \ldots, n + m.$

The methods used in the local properties of signals do not require solving systems of algebraic equations. In these cases, the amount of computation does not depend on the number of grid nodes but is determined only by the degree of the spline. Therefore, it turns out to be much smaller than when constructing interpolation splines, and the equations used to calculate the coefficients give a slight decrease in accuracy.

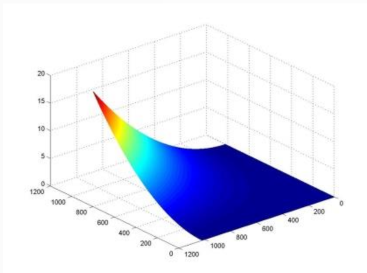
Based on the analysis and research of the properties of spline functions, the so-called local smoothing equations for calculating the coefficients are obtained.

In the presented figures, the red line is the graph of the initial function (analytical functions and experimental data), i.e. f(x), the blue line is the graph of the results of processing the analytical functions and experimental data with one-dimensional cubic basic splines, i.e. S(x). In the following figures shown the results of approximating analytical functions with the number of processed samples N = 64.



Evaluation of signal approximation results

To evaluate the performance of the proposed method, we conducted experiments using a PC with 3.60 GHz CPU



This type of assessment allows us to determine the accuracy of reconstruction of the signal samples that is the quality of recovery.

In conclusion, the development of multidimensional signal processing algorithms and methods based on basis functions is an important and ongoing area of research. By decomposing a signal into a set of basis functions, these methods allow for efficient signal analysis and processing, with applications in fields such as image and video processing, signal compression, and machine learning. The choice of basis functions is critical, as they must be orthogonal or linearly independent and should span the entire signal space. Additionally, optimization of the algorithms is important for efficiency and effectiveness. As technology continues to advance, multidimensional signal processing algorithms and methods will play an increasingly important role in many areas of science and engineering.

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