IDEAL GAZLARDA KVANT STATISTIKASI TAHLILI

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Ushbu maqolada: *ideal gazlarda kvant statistikasi taqsimoti, ideal gaz energiyasi, impulsi, Boze taqsimoti va boshqa parametrlar haqida qisqacha tahlil olib borilgan.*

Kalit so'zlar: *taqsimot funksiya, holatlar zichligi, de-Broyl to'lqin uzunligi, Boze kimyoviy potensiali.*

Kvant mexanikasida zarrachalarning energiyasi ε_k, impulsi p va to'lqin soni k - de-Broyl munosabatlarini hisobga olsak, quyidagicha bog'langan

$$p=\hbar k\,,\quad \varepsilon_p = p^2/2m\,,\qquad \varepsilon_k=\hbar^2k^2/2m \qquad (1)$$

Kvant statistikasiga ko'ra, gaz zarralarining energiya ε_k yoki impuls p yoki to'lqin soni k bo'yicha taqsimot funksiyasi ishlatiladi va quyidagicha yoziladi

$$n_{\varepsilon_k}=\frac{1}{\exp(\frac{\varepsilon_k-\mu}{k_bT})\pm1}\,,\quad n_p=\frac{1}{\exp(\frac{p^2/2m-\mu}{k_bT})\pm1}\,,\quad n_k=\frac{1}{\exp(\frac{\hbar^2k^2/2m-\mu}{k_bT})\pm1} \qquad (2)$$

Bu funksiyalar ma'nosi quyidagicha: n_s,, n_p, n_k, - zarrachaning ε_k energiyali, yoki impulsi p yoki to'lqin soni k bo'lgan holatda yotish (topilish) extimolini bildiradi. (2) formulalarda k_b - Bolsman doimiysi, μ- gazning kimyoviy potensiali deyiladi. Maxrajdagi "+" ishora spinlari $s=1/2$ ga teng bo'lgan Fermi statistikasidagi zarralar uchun (masalan metallardagi elektron gazlar),"-" ishora esa spinlari $s\ =0$ ga teng bo'lgan Boze statistikasidagi zarralar uchun (masalan kristalllardagi fononlar yoki ba'zi gazlar: Geliy He^4, Vodorod molekulali gaz H_2, Neon Ne, Argon Ar va hokazo) uchun ishlatiladi. Quyida biz faqat Boze taqsimot funksiyasining tabiati, ya'ni zarrachaning ε_k energiyali, yoki k holatda yotish (topilish) ehtimolligi funksiyasini o'rganamiz.

$$n_{\varepsilon_k}=\frac{1}{\exp(\frac{\varepsilon_k-\mu}{k_bT})-1}=\frac{1}{\exp(\frac{\hbar^2k^2/2m-\mu}{k_bT})-1} \qquad (3)$$

Ko'rinib turibdiki, kimyoviy potensialning qiymati musbat bo'la olmaydi. Chunki, u holda $\varepsilon_k=0$ energiyali (yoki $k\ =\ 0$ impulsli) holatda zarrachaning topilish ehtimoli manfiy bo'lib qoladi

$$n_{\varepsilon_k} = \frac{1}{\exp(\frac{-\mu}{k_b T}) - 1} < 0 \qquad (4)$$

Bu mumkin emas, chunki n_k – extimollik faqat musbat son bo'lish kerak. Demak, μ ning qiymati faqat nol yoki manfiy qiymatlargagina erishish mumkin $\mu \le 0$ Kimyoviy potensial μ- gaz konsentratsiyasi n va temperatura T ning funksiyasi bo'lib, quyidagi tenglikdan - zarralar sonining normallashtirish tenglamasidan topiladi

$$n = \frac{N}{V} = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} n_{\varepsilon_k} dk_x dk_y dk_z = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{e^{\frac{\varepsilon_k - \mu}{k_b T}} - 1} dk_x dk_y dk_z \qquad (5)$$

Integralni hisoblash uchun Dekart fazosidan sferik koordinatalar fazosiga o'tamiz.

$$n = \frac{N}{V} = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} n_{\varepsilon_k} dk_x dk_y dk_z = \frac{1}{(2\pi)^3} \int_0^{\infty} k^2 dk \int_0^{\pi} \sin\varepsilon d\theta \int_0^{2\pi} d\varphi \frac{1}{e^{\frac{\varepsilon_k - \mu}{k_b T}} - 1} \qquad (6)$$

Integral ostidagi funksiya θ va φ burchaklarga bog'lik emas. Shuning uchun

$$\int_0^{\pi} \sin\theta d\theta = -\int_0^{\pi} d\cos\theta = -\int_1^{-1} dt = \int_{-1}^{+1} dt = 2, \qquad \int_0^{2\pi} d\varphi = 2\pi \qquad (7)$$

Natijada (6) tenglik quyidagicha ko'rinish oladi

$$n = \frac{4\pi}{(2\pi)^3} \int_0^{\infty} k^2 dk \frac{1}{e^{\frac{\varepsilon_k - \mu}{k_b T}} - 1} \qquad (8)$$

Bu inegralni hisoblash uchun k o'zgaruvchidan ε o'zgaruvchiga o'tib olamiz. Buning uchun (1) munosabatlardan foydalanib

$$\varepsilon = \frac{\hbar^2 k^2}{2m}, d\varepsilon = \frac{\hbar^2}{m} kdk, kdk = \frac{md\varepsilon}{\hbar^2}, k^2 dk = \frac{\sqrt{m\varepsilon}}{\hbar}\frac{md\varepsilon}{\hbar^2} = \frac{\sqrt{2}m^{3/2}}{\hbar^3}\varepsilon^{1/2} d\varepsilon$$

(9)

(8) tenglikni quyidagicha yozamiz

$$n = \frac{4\pi}{(2\pi)^3} \frac{\sqrt{2}m^{3/2}}{\hbar^3} \int_0^{\infty} \frac{\varepsilon^{1/2} d\varepsilon}{e^{\frac{\varepsilon_k - \mu}{k_b T}} - 1} \qquad (10)$$

Bu integralni analitik kurinishda hisoblab bo'lmaydi,faqat sonli usulda hisoblash mumkin. (10) tenglikdan ko'rinadiki konstentrastiya n berilganda (fikrlab olamiz, ya'ni $n = const$) temperatura T ning o'zgarishi μ ning o'zgarishiga olib keladi. (4) dan ma'lumki $\mu \le 0$. Faraz qilaylik biror temperatura T_0 da $\mu = 0$ bo'lsin. Shu temperatura T_0 ni topish uchun (10) tenglikni quyidagicha yozamiz

$$n = \frac{4\pi}{(2\pi)^3} \frac{\sqrt{2}m^{3/2}}{\hbar^3} \int_0^{\infty} \frac{\varepsilon^{1/2} d\varepsilon}{e^{\frac{\varepsilon_k - \mu}{k_b T}} - 1} \qquad (11)$$

Bu integralni analitik ko'rinishda hisoblanadi, va quyidagini olamiz

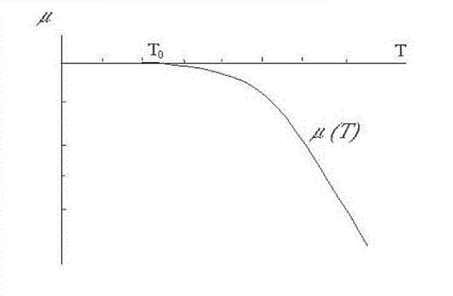
$$n = \frac{4\pi}{(2\pi)^3}\frac{\sqrt{2}m^{3/2}}{\hbar^3}(k_b T_0)^{3/2}\int_0^\infty \frac{x^{1/2}dx}{e^x - 1} = \frac{4\pi}{(2\pi)^3}\frac{\sqrt{2}m^{3/2}}{\hbar^3}(k_b T_0)^{3/2}\Gamma(\frac{3}{2})\xi(\frac{3}{2})$$

(12)

Bu yerda $\Gamma(3/2) = \pi^{1/2}/2$ va $\xi(3/2) = 2.612$. U xolda (12) dan T_0 temperatura uchun quyidagini olamiz

$$k_b T_o = \frac{3.31\hbar^2 n^{2/3}}{m} \qquad (13)$$

Demak, konstentrastiyaning n berilgan ma'lum bir qiymatida shunday bir T_0 temperatura mavjud ekanki, bu temperaturada ideal Boze – gaz kimyoviy potenstiali nolga teng ekan. Barcha temperatura intervalida $\mu(T)$ bog'lanishi (2.10) dan sonli usulda hisoblangan natijasi quyidagicha ko'rinishda bo'ladi (1-Rasm)



1-rasm. Ideal Boze gaz kimyoviy potensialining temperaturaga bog'lanishi.

Demak, ideal Boze - gaz kimyoviy potensiali temperatura ortishi bilan - ∞ da ga qarab o'sar ekan. Temperatura kamaysa ma'lum bir T_0 da nolga teng bo'lib to $T = 0$ gacha doimo nol bo'lib qolar ekan. Lekin (10) tenglikdan ko'rinadiki konstentrastiya $n = const$ (fiksirlangan) bo'lgani uchun temperaturani $T < T_0$ tomonda kamayishida integralning qiymati ham kamaya boshlash kerak, chunki μ($T<T_0$) =0. Bu holda (10) tenglik bajarilib turishi uchun uning o'ng tomoniga integralning kamayib borishini kompensastiyalovchi "nimadir" qo'shish kerak

$$n = n_0 + \frac{4\pi}{(2\pi)^3}\frac{\sqrt{2}m^{3/2}}{\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}d\varepsilon}{e^{\frac{\varepsilon}{k_b T}} - 1}, T < T_0 \qquad (2.14)$$

$T < T_0$ da $\mu(T < T_0) = 0$ bo'lgani sababli Boze taqsimot funksiyasi (3) quyidagicha yoziladi

$$n_k = \frac{1}{\exp(\frac{\varepsilon_k}{k_b T})} = \frac{1}{\exp(\frac{\hbar^2 k^2 / 2m}{k_b T}) - 1}, T < T_0 \qquad (2.15)$$

Demak, $T < T_0$ da $k = 0$ holatda (eng quyi holat) zarrachaning topilish ehtimoli ∞ ga teng ekan, $n_{k-0} = \infty$.

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