# QISQA KO`PAYTIRISH FORMULALARI VA NYUTON BINOMI

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**Annotatsiya:** *Ushbu maqolada Matematika fanining asosiy yordamchisi hisoblanadi. Bu maqolani avzali tomonlari shundaki qisqa ko`paytirish formulasi va Nyuton binomi keltirib o`tilgan. Nyuton binomi haqida yangi elementlarga ega bo`lishga ushbu maqola yordam beradi.*

**Kalit so'zlar**: *Nyuton binomi, qisqa ko`paytirish, Paskal uchburchagi,*

Quyidagi formulalarga qiska ko`paytirish formulalari deyiladi.

1. $(a+b)^2 = a^2 + 2ab + b^2$ -ikki had yig`indisining kvadrati;
2. $(a-b)^2 = a^2 - 2ab + b^2$ -ikki had ayirmasining kvadrati;

**3.** $a^2 - b^2 = (a-b)(a+b)$ -ikki had kvadratlarining ayirmasi;

**4.** $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ -ikki had kublarining yig`indisi;

**5.** $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ -ikki had kublarining ayirmasi;

**6.** $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ -ikki had yig`indisining kubi;

**7.** $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ -ikki had ayirmasining kubi.

Keltirilgan 1-7 formulalar ko`phadni ko`phadga ko`paytirish qoidasiga asosan oson isbotlanadi. Misol uchun 1;5;7 -formulalarning isbotini keltiramiz:

1. $(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

5. $(a-b)(a^2 + ab + b^2) = a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 = a^3 - b^3$

7. $$(a-b)^3=(a-b)(a-b)^2=(a-b)(a^2-2ab+b^2)=$$

$$a^3-2a^2b+ab^2-a^2b+2ab^2-b^3=$$

$$=a^3-3a^2b+3ab^2-b^3$$

Endi qisqa ko`paytirish formulalaridan 1 va 6 formulalarni taxlil qilamiz:

1. $(a+b)^2=a^2+2ab+b^2$ bu formulaning o`ng tomoniga e`tibor bersak, $a^2b^0, a^1b^1, a^0b^2$ hadlar hosil bo`lishida $a$ ning darajasi pasayib, b ning darajasi oshib borayotganini ko`ramiz.

2. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$$(a+b)^4=(a+b)(a+b)^3=(a+b)(a^3+3a^2b+3ab^2+b^3)=$$

$$=a^4+4a^3b+6a^2b^2+4ab^3+b^4, \text{яъни\_}(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

Xuddi shu usul bilan $(a+b)^5$; $(a+b)^6;...;(a+b)^n$ uchun ikki had yig`indisini darajaga ko`tarish formulasini hosil qilish mumkin. Bunda koeffitsientlar «Paskal uchburchagi» deb ataluvchi jadvaldan olinadi.

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1 | | | | | | | |
| | | | | | | 1 | | 1 | | | | | | |
| | | | | | 1 | | 2 | | 1 | | | | | |
| | | | | 1 | | 3 | | 3 | | 1 | | | | |
| | | | 1 | | 4 | | 6 | | 4 | | 1 | | | |
| | | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | | |
| | 1 | | 6 | | 15 | | 20 | | 15 | | 6 | | 1 | |
| 1 | | 7 | | 21 | | 35 | | 35 | | 21 | | 7 | | 1 |

Misol

$$(a+b)^6=a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$$

Agar $(a+b)^{100}$ ni ochib chiqish lozim bo`lsa, yoyilmada 101 ta had hosil bo`ladi va bu yoyilma koeffitsientlarini Paskal jadvali buyicha hisoblash qiyin bo`ladi. Shu sababli $(a+b)^n$ ni ko`phadga yoyganda hosil bo`ladigan $a^k\cdot b^{n-k}$ had oldidagi koeffitsient $C_n^k$ -dan, ya`ni n elementdan $k$ tadan qilib tuzilgan gruppalashlar sonidan iborat ekanligi isbotlangan, bu erda $C_n^k=\dfrac{n!}{k!\cdot(n-k)!}$, $n!=1\cdot 2\cdot ...\cdot n$.

Misol: $C_5^2;\ C_9^5;\ C_{12}^7$ hisoblansin:

$$C_5^2=\frac{5!}{2!\cdot(5-3)!}=\frac{2!\cdot3\cdot4\cdot5}{2!\cdot2!}=\frac{3\cdot4\cdot5}{2}=30; \quad C_9^5=\frac{9!}{5!\cdot4!}=\frac{5!\cdot6\cdot7\cdot8\cdot9}{5!\cdot1\cdot2\cdot3\cdot4}=125$$

$$C_{12}^7=\frac{12!}{7!\cdot(12-7)!}=\frac{7!\cdot8\cdot9\cdot10\cdot11\cdot12}{7!\cdot1\cdot2\cdot3\cdot4\cdot5}=\frac{8\cdot3\cdot3\cdot5\cdot2\cdot11\cdot12}{8\cdot3\cdot5}=6\cdot11\cdot12=792$$

Endi umumiy holda matematik induktsiya usuli yordamida N`yuton binomi deb ataluvchi quyidagi formulani isbotlaymiz:

$$(a+b)^n=C_n^0a^n+C_n^1a^{n-1}b+C_n^2a^{n-2}b^2+...+C_n^ka^{n-k}b^k+...C_n^nb^n \qquad (1)$$

Bu erda $C_n^k$-lar binom koeffitsientlari deyiladi va quyidagicha hisoblanadi.

$C_n^k=\dfrac{n!}{k!\cdot(n-k)!}$, $\quad C_n^0=C_n^n=1$, n=1 bo`lsa,

$$(a+b)^1=C_1^0a^1+C_1^1b^1=a+b$$

Endi (1) formula $n=k$ bo`lganda o`rinli deb, uning $n=k+1$ bo`lganda ham o`rinli ekanligini isbotlaymiz, ya`ni

$$(a+b)^k=C_k^0a^k+C_k^1a^{k-1}b+...+C_k^la^{k-l}b^l+...+C_k^{k-1}ab^{k-1}+C_k^kb^k \qquad (2)$$

bo`lganda

$$(a+b)^{k+1}=C_k^0a^{k+1}+C_k^1a^kb+...+C_k^la^{k-l}b^l+...+C_k^{k-1}ab^k+C_k^kb^{k+1} \quad (3)$$

tenglikning o`rinli ekanligini isbotlaymiz:

$$(a+b)^{k+1}=(a+b)(a+b)^k=(a+b)(C_k^0a^k+C_k^1a^{k-1}b+C_k^2a^{k-2}b^2+...+C_k^la^{k-l}b^l+...+$$
$$+C_k^kb^k)=C_k^0a^{k+1}+C_k^1a^kb+C_k^2a^{k-1}b^2+...+C_k^la^{k-l+1}b^l+C_k^kab^k+C_k^0a^kb+C_k^1a^{k-1}b^2+...+$$
$$+C_k^{k-1}ab^k+C_k^kb^{k+1} \qquad (4) \quad bundan\ esa$$

$$(a+b)^{k+1}=C_k^0a^{k+1}+(C_k^1+C_k^0)a^kb+(C_k^2+C_k^1)a^kb^2...+(C_k^l+C_k^{1-1})a^{k-l+1}b^l+...+$$
$$+(C_k^k+C_k^{k-1})ab^k+C_k^kb^{k+1} \qquad (5)$$

ravshanki,

$$C_k^0=1=C_{k+1}^0, \quad C_k^k=1=C_{k+1}^{k+1}, \quad C_n^{m+1}+C_n^m=\frac{n!}{(m+1)!(n-m-1)!}+\frac{n!}{m!(n-m)!}=$$

$$=\frac{n!}{m!(n-m-1)!}\cdot\left(\frac{1}{m+1}+\frac{1}{n-m}\right)=\frac{n!}{m!(n-m-1)!}\cdot\frac{n-m+m+1}{(m+1)(n-m)}=\frac{(n+1)!}{(m+1)!(n-m)!}=C_{n+1}^{m+1}$$

Oxirgi tengliklarni hisobga olsak, (5) dan (3) tenglikni o`rinli ekanligini topamiz.

Endi matematik induksiya usuli bilan (5) formulani umumlashtiramiz, ya`ni

$$A^n-B^n=(A-B)(A^{n-1}+A^{n-2}B+A^{n-3}B^2+...+AB^{n-2}+B^{n-1}) \qquad (7)$$

formulani isbotlaymiz:

$n=2 \quad bo'lsa, \qquad A^2-B^2=(A-B)(A+B)$

(7) tenglikni $n=k$ uchun to`g`ri deb, $n=k+1$ uchun isbotlaymiz, ya`ni

$$(A-B)(A^k+A^{k-1}B+A^{k-2}B^2+...+AB^{k-1}+B^k)=A^{k+1}+B^{k+1} \quad (8)$$

ekanini isbotlaymiz.

$$(A-B)(A^k+A^{k+1}B+A^{k-2}B^2+...+AB^{k-1}+B^k)=(A-B)(A^k+A^{k+1}B+A^{k-2}B^2+...+AB^{k-1}+B^k)+(A+B)B^k=$$
$$=(A-B)(A^{k-1}+A^{k-2}B+A^{k-3}B^2+...+B^{k-1})A+(A-B)B^k=(A^k-B^k)A+(A-B)B^k=A^{k+1}-AB^k+AB^k-B^{k+1}=$$
$$=A^{k+1}-B^{k+1}$$ shuni isbot qilish tal ab etilgan edi.

N`yuton binomi formulasini ba`zi bir xossalarini o`rganamiz:

$$(x+a)^n=C_n^0x^na^0+C_n^1x^{n-1}a+C_n^2x^{n-2}a^2+C_n^mx^{n-m}a^m+...+C_n^nx^0a^n \quad (1)$$

1. $C_n^0,\ C_n^1,\ C_n^2,...,C_n^n$ larga binominal koeffitsientlar deyiladi.
2. N`yuton binomi quyidagi xosalarga ega:
3. N`yuton binomida hadlar soni n-darajadan bittaga ziyod, ya`ni n+1 ta.
4. Unda qatnashayotgan birhadlarda x bilan a ning darajalari yig`indisi $(n-m)+m=n$ ga teng.
5. Uning umumiy hadi $C_n^mx^{n-m}a^m$ ga teng bo`lib, $T_{m+1}=C_n^mx^{n-m}a^m,\ (m=0,1,2,...n)$ ko`rinishda belgilanadi.
6. N`yuton binominig oxirgi hadlaridan teng uzoqlikda turgan hadlar o`zaro teng, ya`ni $C_n^m=C_n^{n-m}$ ва $C_n^0=C_n^n=1;\_C_n^1=C_n^{n-1}=\frac{n!}{1!(n-1)!}=n,...,$
7. N`yuton binomining barcha binomial koeffitsientlari yig`indisi $2^n$ ga teng. Haqiqatdan (1) formulada $x=a=1$ bўlsa, $2^n=C_n^0+C_n^1+...+C_n^m+...+C_n^n$
8. N`yuton binomida juft va toq o`rnida turgan binomial koeffitsientlar yig`indisi o`zaro teng va qiymati $2^{n-1}$ ga teng, ya`ni $C_n^0+C_n^2+C_n^4+...=C_n^1+C_n^3+C_n^5+...=2^{n-1}$

**FOYDALANGAN ADABIYOTLAR ;**


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